Economies of Scope from Shared Inputs[*]

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Abstract

The prevailing explanation for large, multi-product firms is economies of scope driven by shared inputs across production lines. Using data from the Federal Trade Commission's Line of Business Surveys, which detail both line-specific and shared inputs, we show that US manufacturing firms report substantial shared inputs for both capital and management/marketing expenses. The use of shared inputs is correlated with firm size and scope. We estimate a nested CES production function between private inputs and shared inputs, which are substitutes with an elasticity of substitution of 2.5. Shared inputs provide significant economies of scope: reducing shared inputs by 50% would decrease output by 3.6% for the average multi-product firm. Finally, average merger synergies from greater economies of scope in merger simulations are 1.6% to 2.6%.

Keywords: production function, economies of scope, multiproduct firms, productivity

JEL Codes: D24, L23, L40, L60

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1 Introduction

Large, multi-product firms are responsible for a substantial share of US output and the R&D spending contributing to productivity growth.[1] The dominant explanation for the existence of such firms is economies of scope, defined as cost savings resulting from a firm producing multiple products (Baumol et al., 1982; Panzar and Willig, 1981). Panzar and Willig (1981) formally show that economies of scope imply the existence of shareable inputs across different production lines, such as physical capital, knowledge, management, and marketing.

Despite this theoretical result, however, data limitations have caused the recent empirical literature examining multi-output production to largely ignore such shared inputs.[2] In most cases, at best only output data may be available at the product level, but input data is only available at the firm level. Even if product-level input data are observed, identifying which inputs are shared across production lines is challenging (Panzar and Willig, 1981).

Researchers have responded to these data limitations by taking two basic approaches. First, one could fully allocate inputs to different production lines (De Loecker et al., 2016; Gong and Sickles, 2021; Itoga, 2019; Orr, 2022; Valmari, 2023), which implicitly assumes no common inputs or economies of scope arising from such inputs. Alternatively, one could estimate a transformation function from firm-level inputs to multiple outputs (Dhyne et al., 2022; Diewert, 1973; Grieco and McDevitt, 2017; Lau, 1976; Maican and Orth, 2021; Malikov and Lien, 2021); however, such a transformation function is likely to be firm-specific for large firms that operate in unique sets of business lines.[3]

In this article, we examine how inputs common across product lines affect economies of scope using microdata for large US manufacturing firms in the 1970s from the FTC's Line of Business Surveys (Ravenscraft and Wagner III, 1991), which we describe in Section 2. The FTC Line of Business surveys are unique for two reasons: they have data on both revenue and inputs at the line

[1]For example, Gabaix (2011) document that the sales of the top 100 US firms represent 29% of US GDP on average and account for one-third of the volatility in output growth; Anderson (2024) finds that firms with more than 20,000 employees constitute 38% of all corporate R&D spending, and above 10,000 employees 55% of corporate R&D spending, in 2021.

[2]We use the terms "common", "shared/shareable", and "public" interchangeably to refer to inputs that are not specific to a line of business.

[3]Most of these papers have examined a setting with a small number of outputs (such as beef and dairy milk from cows or quality and quantity from dialysis). However, Dhyne et al. (2023) examine the assumptions required to estimate transformation functions with many outputs.

of business level, and ask how much inputs, such as capital and management/marketing expenses, were specific to a given line of business (Nichols, 1989).[4] In other words, the data identifies how inputs are allocated across products and which inputs are publicly shared across products within firms. Thus, our data allows us to estimate production functions at the firm-line-of-business level while allowing for common inputs. Then, we can use those estimates to assess the importance of common inputs in generating economies of scope.

We begin by documenting stylized facts about the two common inputs we observe—capital and management/marketing expenses—in Section 3. First, firms report significant common inputs; two-thirds of firms and three-quarters of firm-lines of business report positive values of both capital and management. For the average line of business with positive shared input, the shared input is 20% larger than the private input for capital and 270% larger for management. In addition, as Argente et al. (2020) predict, we find that the ratio of shared input to private input is positively associated with firm size and scope.

Given these facts, in Section 4 we model output as a nested CES production function between a private input and common input, where both are Cobb-Douglas functions of sub-inputs. The key parameters in this production function that determine the degree of economies of scope are the elasticity of substitution between the private input and common input and the distribution parameter weighting these inputs. Because we only observe revenue, not output, we derive the revenue production function after assuming a CES demand function.

We identify the parameters of this production by modifying the approach of Gandhi et al. (2020) in Section 5. For firms reporting positive common inputs, we identify the production function using moments from firms' input share equations based on first-order conditions with respect to flexible inputs. In addition, we develop dynamic panel moments using Markov assumptions on the stochastic process of the unobserved productivity terms and demand shocks. Our product-level input/output data allow us to apply Gandhi et al. (2020)'s single-output production function estimation method to estimate multi-output production functions.

In Section 6, we estimate the nested CES production and find that private inputs and public inputs are substitutes, with an elasticity of substitution of 2.5 and a distribution weight of 0.05

[4] Nichols (1989) provides an earlier attempt to study economies of scope with the FTC Line of Business Survey data using an ordinary least squares-based correlation analysis. We take a structural approach and estimate a production function with common inputs.

on shared inputs. Shared inputs matter for revenue; we estimate average (median) line of business level revenue elasticities of 0.02 (0.01) for shared capital and 0.05 (0.03) for management and marketing expenses. We also find substantially higher aggregate revenue elasticities from increasing common inputs; for the median firm, the aggregate revenue elasticity is 0.10 for capital and 0.11 for management.

Using our estimates, we estimate productivity at the firm-line of business level. Most productivity differences occur within firms, not across firms; through a variance decomposition, we estimate that about three-quarters of productivity differences occur within firms compared to one-quarter across firms.

To measure the importance of economies of scope, we examine counterfactual scenarios reducing the shared input in Section 7. We estimate that reducing the common input by 50% reduces firms' revenue by 3.6% on average. We also show that firms with a higher number of lines of business are subject to a larger revenue loss; a 50% reduction in the common input reduces firms with 2 to 3 lines of business by 2.5%, compared to 4.2% for firms with 10 or more lines of business.

Finally, we examine merger synergies stemming from economies of scope through a merger simulation exercise. We simulate all possible mergers of firms with no overlap in production lines and assume each merger allows firms to pool their public resources through either the maximum or sum of their common inputs. We find modest merger synergies from economies of scope: for the average merger, greater economies of scope increase total revenue by 1.6% to 2.6%.

Our paper complements Khmelnitskaya et al. (2024), which estimates economies of scope from shared inputs using demand-based estimates of marginal costs rather than production data. The authors find that shutting down economies of scope would increase marginal costs and prices substantially in the US beer industry. They also show how to adapt merger simulations to economies of scale and scope, and show that the Miller-Coors merger would provide significant cost savings from greater scope economies. In addition, Cairncross et al. (2024) show that product-level markups are not identified, and firm-level markups are identified, given shared inputs in production as well as within-firm productivity differences across products; we document the presence of both.

A recent literature in macroeconomics also examines shared inputs and economies of scope. Ding (2023) build a model where shared inputs allow the firm to develop knowledge, which can be then allocated across industries, and uses the model to quantify aggregate economies of scope from knowledge inputs in US manufacturing. Boehm et al. (2022) estimate the economies of scope arising from factor-biased productivities that are jointly used across lines and find that economies of scope are important determinants of product market entry.[5]

Our paper is most closely related to Argente et al. (2020), who develops a model in which firm productivity depends on a CES function of shareable and private inputs. The key parameter in their model is the elasticity between the shareable and private input. Argente et al. (2020) argue that the empirically relevant case is when the shareable input and private input are substitutes, as we find. Their model predicts that the shareable input to private input ratio will be positively correlated with size and scope and that firms with larger size or scope will be more sensitive to demand shocks. Finally, they show how economies of scope from shared inputs can amplify the effects of greater productivity on firm revenue.

Finally, the recent debate over antitrust policy has led policymakers to re-consider whether to reinstate the Line of Business surveys, which were discontinued in the Reagan-era antitrust reforms. Senator Amy Klobuchar has recently argued in favor of restarting this data collection program in her recent book *Antitrust* (Klobuchar, 2021).[6] We show that the design of the FTC's Line of Business Surveys provides valuable information not available from existing data sources.

2 Data

2.1 Background

In the 1970s, the FTC developed a program to collect disaggregated data on revenue and costs from the largest manufacturing firms in the US. The FTC piloted the survey in 1973 and then ran four annual waves from 1974 to 1977 (U.S. Federal Trade Commission, 1985). This data effort experienced considerable headwinds, as hundreds of corporations sued to stop the data collection. While the FTC won in court (Whipple, 1979), the Government Accountability Office (GAO) asked

[5]In related work, An et al. (2019) examine the elasticity of substitution between private and public capital and find that private inputs are complementary with public capital. However, they define public capital as public sector (i.e., government) capital rather than capital owned by firms and employed across different production lines, as in this paper.

[6]She writes *"The FTC used to collect industry data on lines of business in an effort to make sure particular sectors did not become too concentrated, and antitrust officials today also have a need to get accurate information so that they can closely monitor industries for monopoly power and consolidation. Although the data collection program was stopped in the mid-1980s, if antitrust agencies are adequately funded, they will be better able to use modern-day technology to effectively track anticompetitive or exclusionary conduct."*

the FTC to evaluate the benefits and costs of the surveys. Data collection was paused pending this cost-benefit analysis. The FTC eventually concluded that the costs exceeded the benefits in 1984, so the program was discontinued.

The FTC asked large manufacturing firms to provide data at the "line of business" level, which was defined differently from the Standard Industrial Classification (SIC) codes to better reflect the economic realities and operations of diversified firms and their competition. The FTC developed 289 lines of business.[7] For example, in the glass industry, flat glass (SIC 321), glass containers (SIC 3221), pressed and blown glass not classified elsewhere (SIC 3229), and products made from purchased glass (SIC 323) are each considered distinct lines of business. In addition, the data include information on 14 non-manufacturing lines of businesses at a roughly one-digit SIC level of aggregation (e.g., construction or retail trade), which we do not use in this study.

2.2 Information in the Data

Table 1 reports the number of firms, total lines of business, and manufacturing lines of business in the data. Each annual wave has between 436 and 469 firms in the sample. There are between 4,291 and 4,650 lines of business in total. Most lines of business are in the manufacturing category. On average, firms operate in about 10 lines of business, of which 7 to 8 are in manufacturing.

Table 1: Number of Firms and Lines of Business Per Year

Year	Firms	Lines of Business	Manufacturing Lines of Business
1974	436	4,291	3,383
1975	469	4,507	3,536
1976	466	4,572	3,598
1977	456	4,650	3,693

We observe a high level of heterogeneity in the number of lines of business that firms operate. Figure 1 depicts the distribution of manufacturing lines of business across all firms and years. This distribution is quite skewed. While the modal firm has 5 lines of business and the median firm has 6 lines of business, 25% of firms have 10 lines of business or more, and 5% have over 20 lines of business.

[7] See U.S. Federal Trade Commission (1985) Appendix E for the full list of FTC lines of business and corresponding SIC codes.

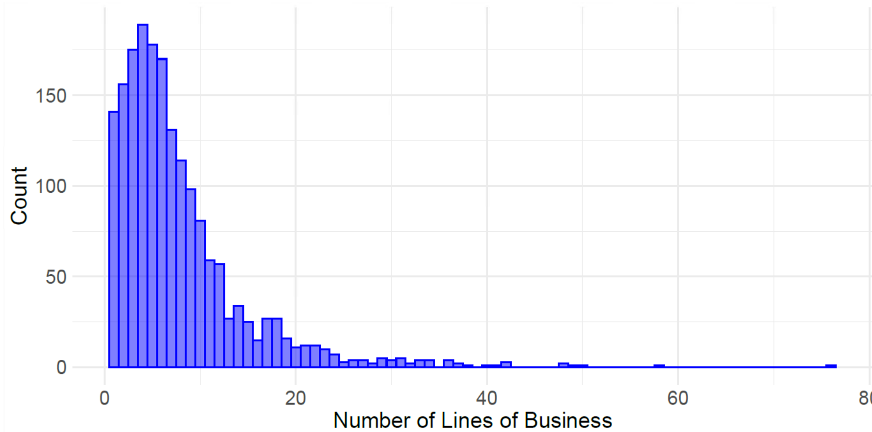

Figure 1: Distribution of Manufacturing Lines of Business

The dataset includes information on the standard productivity inputs and outputs—sales, materials, payroll, and capital (net plant, property, and equipment)—at the line-of-business level. In addition, it has information on three categories of additional expenses at the line of business level—advertising, other selling, and general/administrative expenses—which we combine and refer to as management/marketing expenses or management for short.[8]

For both capital and management, the survey distinguished between assets or expenditures "traceable" to the line of business and those that are not specific to a line of business. The FTC defined "traceable" as follows[9]:

> Those costs and assets which a company can directly attribute to a line of business or which can be assigned to a line of business by use of a reasonable allocation method developed on the basis of operating level realities.

We use this distinction to separate common inputs across the firm's lines of business from inputs specific to a given line of business. One limitation of this approach is that we cannot separate inputs common to all lines of business from those that might be shared by a subset of production lines. In addition, the survey distinguished revenue from sales to outside parties from transfers to different lines of business of the firm.

[8] Although we have data on revenues and input expenditures, we lack information on physical quantities of outputs and inputs, as well as their prices.

[9] The FTC referred firms to the Financial Accounting Standards Board's Standard No. 14 (FAS 14). For expenses, traceable could be compared to the terms "directly traceable" and "allocated on a reasonable basis" in FAS 14, and non-traceable expenses to "general corporate expenses" in FAS 14. For assets, traceable could be compared to the term "identifiable" in FAS 14 and non-traceable assets to the term "assets maintained for general corporate purposes."

While our sample has less than 500 firms per year, these firms collectively represent a significant share of the manufacturing industry. When compared to the 1977 NBER Productivity Database (Bartelsman and Gray, 1996; Becker et al., 2021), the firms in our data account for 47–53% of the manufacturing revenue, 49% of materials, and 53% of payroll. The firms also account for 73–84% of the manufacturing gross capital in the 1977 Census of Manufactures (U.S. Department of Commerce, 1981).[10]

We clean the data by dropping all non-manufacturing lines of business and all observations with zero or negative records for sales, payroll, materials, traceable capital, and traceable management/marketing expenses. The latter restriction removes about 6% of all observations in the data.

3 Stylized Facts

The most unique feature of the FTC's Line of Business Surveys is that they asked firms to report how much of certain inputs were "traceable" to a given line of business. In this section, we examine the distribution of shared inputs for two such inputs—capital and management/marketing expenses—as well as how these inputs vary by the size and scope of the firm. For simplicity, we also refer to the combination of management and marketing expenses as "management."

In our data, most firms report positive shared inputs for both capital and management. Across firms, 75% report positive shared management/marketing expenses, 70% positive shared capital, and 66% positive amounts of both shared inputs.[11]

We next examine the *scalability ratio* (Argente et al., 2020), defined as the ratio of the shareable input to the private input for a given firm and product line.[12] Argente et al. (2020) show that the scalability ratio provides the right sufficient statistic for how much the firm's input (in their model, expertise) can be applied across its products.

[10]For revenue calculations, the lower bound is based on revenue from external transactions, while the upper bound includes within-firm transfers. For capital calculations, the lower bound is based on only traceable capital and the upper bound also includes non-traceable capital. When constructing capital inputs, we use the net capital stock and not gross capital stock. However, it was simplest to compare gross capital stocks across databases.

[11]Firms with more lines of business are more likely to report positive common inputs, as we show below. Thus, examining firm-lines of business, we find 83% of lines of business have positive shared management, 78% positive shared capital, and 74% positive values of both inputs.

[12]That is, scalability ratio of input X is calculated as $\frac{X_i^C}{X_{ij}^P}$, where X_i^C is the shareable input for firm i and X_{ij}^P is the private input for firm i and line of business j.

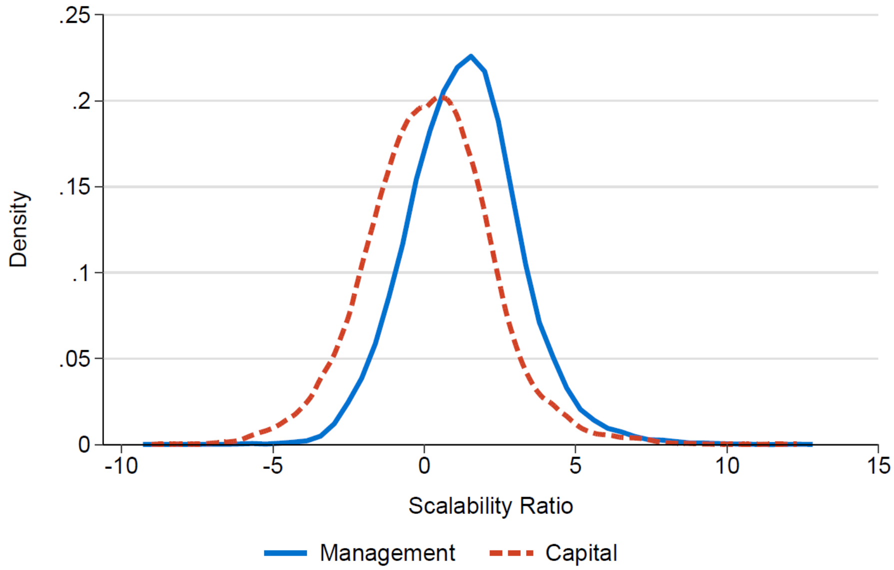

Figure 2: Density of Scalability Ratio

In Figure 2, we depict the density of the log scalability ratio for each input across firms and business lines given positive amounts of the given shareable input. Both densities are approximately symmetric, with little difference between the median and the mean. For capital, the median firm-line of business has 20% (log ratio of 0.18) more shareable capital than private capital; for management expenses, the median firm-line of business has 270% (log ratio of 1.31) higher shareable management than private management expenses. Thus, management inputs appear more scalable than capital.

Argente et al. (2020) predict that firms with greater size and scope should have a higher scalability ratio (i.e., greater amounts of scalable input to private input). We test this prediction using the cross-section of our data, defining size as the revenue of the firm and scope as the number of lines of business of the firm. We then estimate the following regression equation:

$$\log(S_{ij}) = \beta_0 + \beta_1 \log(Size_i) + \beta_2 \log(Scope_i) + \gamma X_i + \epsilon_{ij}, \tag{1}$$

where i and j represent firm and line of business, respectively; S_{ij} is the scalability ratio; X_i represents additional controls for the technological sophistication of the firm—the firm's the capital-to-labor cost ratio and the R&D-to-sales ratio.

Table 2 displays the estimation results of (1). The first four columns examine management expenses, and the last four columns examine capital; within each set of columns, the first column examines size alone, the second column examines scope alone, the third column both size and scope, and the fourth column includes the additional controls.

Table 2: Relationship between the Scalability Ratio and Firm Size and Scope

	Management				Capital			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Size	0.44		0.30	0.13	0.48		0.25	0.19
	(0.06)		(0.07)	(0.07)	(0.08)		(0.09)	(0.09)
Scope		0.60	0.39	0.58		0.79	0.63	0.71
		(0.11)	(0.12)	(0.11)		(0.12)	(0.14)	(0.14)
Controls	No	No	No	Yes	No	No	No	Yes
R^2	0.06	0.05	0.07	0.10	0.05	0.08	0.09	0.09

Notes: Table reports the estimates of regression equation (1). Standard errors are clustered at the firm level. Additional controls include the capital-to-labor cost ratio and the R&D-to-sales ratio.

We find robust evidence that the scalability ratio is positively associated with both size and scope, with stronger estimates for scope compared to size.[13] For example, specification (3) finds that a 10% increase in size (resp. scope) is associated with an increase in the scalability ratio for management by 3.0% (resp. 3.9%). The positive associations between the scalability ratio and firms' size and scope remain even after controlling for the firms' technological sophistication, as shown in column (4). We find the same patterns for both management expenses and capital. Thus, our estimation results are consistent with Argente et al. (2020)'s predictions that size and scope positively correlate with scalability ratio.

Finally, we find that the scalability ratio for management and capital are positively correlated. An increase in the scalability ratio for capital by 10% is associated with a 5.2% (standard error: 0.24%) increase in the scalability ratio for management. Similarly, firms with positive shareable input for capital are also more likely to have positive shareable input for management. Having a positive shareable input for capital is associated with a 55% higher probability (standard error:

[13]We also examine how the presence of any shared input relates to firm size and scope. We find that shared inputs are positively correlated with scope but not size. To test this, we replace the scalability ratio in equation (1) with an indicator for positive shared input and include both size and scope as variables. A 10% increase in scope is associated with a statistically significant 0.80 percentage point increase (standard error: 0.28) in the likelihood of positive shared management and a 0.84 percentage point increase (standard error: 0.31) in the likelihood of positive shared capital. In contrast, a 10% increase in firm size has no effect on shared management (0.00, standard error: 0.18) and leads to a small, statistically insignificant decrease of 0.21 percentage points (standard error: 0.25) in the probability of positive shared capital.

5.5%) of having positive shareable input for management.

4 Model

In this section, we assume a nested CES production function between indices of private inputs and shared inputs and then build a revenue production function given CES demand.

4.1 Quantity Production Function

Firms produce via the following production function:

$$Y_{jt} = A_{jt} F(H_{jt}, C_{jt}; \theta), \tag{2}$$

where j indexes the line of business[14], and t the year. Y_{jt} represents physical output. A_{jt} is Hicks-neutral productivity. H_{jt} represents an index of private inputs, and C_{jt} represents an index of shareable or public inputs such that $C_{jt} = C_{j't}$ for all products j and j' being produced by the same firm. Finally, $F_{jt} \equiv F(H_{jt}, C_{jt}; \theta)$ is a parametric function that relates the inputs to physical output. The production function (2) exhibits economies of scope if $\partial F_{jt}/\partial C_{jt} > 0$ and $C_{jt} > 0$ (Panzar and Willig, 1981).

For estimation, we further assume that the production function takes a nested CES form

$$F(H_{jt}, C_{jt}) = (\alpha H_{jt}^{\rho} + (1-\alpha) C_{jt}^{\rho})^{\frac{\gamma}{\rho}}, \tag{3}$$

where α is the distribution parameter that represents the importance of H_{jt} relative to C_{jt}, and $\rho \equiv \frac{\sigma - 1}{\sigma}$ with σ representing the elasticity of substitution.[15] The production function exhibits economies of scope when $\alpha > 0$. We construct the $\tilde{H}_{jt}$ and $\tilde{C}_{jt}$ as the following Cobb Douglas

[14] We use "product" and "line of business" interchangeably. However, lines of business are comparable to SIC industries and may encompass many market products classified under a single SIC code.

[15] $\rho > 0$ (resp. $\rho < 0$) indicates that the inputs are gross substitutes (resp. complements). The CES function includes three special cases: (i) if $\rho \to 0$ ($\sigma \to 1$), then the elasticity of substitution is fixed at unity, and $Y_{jt} = A_{jt}(H_{jt}^{\alpha} C_{jt}^{1-\alpha})^{\gamma}$; (ii) if $\rho \to -\infty$ ($\sigma \to 0$), then the inputs are perfect complements and $Y_{jt} = A_{jt} \min\{H_{jt}, C_{jt}\}^{\gamma}$; (iii) if $\rho \to 1$ ($\sigma \to \infty$), then the inputs are perfect substitutes and $Y_{jt} = A_{jt}[\alpha H_{jt} + (1-\alpha) C_{jt}]^{\gamma}$.

indices:

$$s \qquad \begin{aligned} H_{jt} &= M_{jt}^{\beta_m} L_{jt}^{\beta_l} K_{jt}^{\beta_k} E_{jt}^{\beta_e} \\ C_{jt} &= \mathcal{K}_{jt}^{\delta} \mathcal{E}_{jt}^{1-\delta}, \end{aligned} \tag{4}$$

where M_{jt}, L_{jt}, K_{jt}, and E_{jt} represent private material, labor, capital, and management expenses, and $\mathcal{K}_{jt}$ and $\mathcal{E}_{jt}$ represent public capital and management expenses.

This nested CES production function between private and public inputs is similar to the production function in Argente et al. (2020), except that in our model both private and public inputs are indices of multiple subinputs.[16] Our model generalizes the Cobb-Douglas production function specification assumed by Cairncross et al. (2024) and Khmelnitskaya et al. (2024), which builds on Baumol et al. (1982)), by allowing for a more flexible relationship between private and public inputs.

4.2 Productivity Shock

We specify the Hicks-neutral productivity shock as $A_{jt} \equiv \exp(\omega_{jt} + \varepsilon_{jt})$, where ω_{jt} is the persistent productivity shock, known to the firm before making its period t decision, and ε_{jt} is the independently and identically distributed ex-post productivity shock realized only after period t decisions are made. Let $\mathcal{I}_{jt}$ denote the information set of product j's producer when making period t decisions on inputs. Then, by definition, $\omega_{jt} \in \mathcal{I}_{jt}$ whereas $\varepsilon_{jt} \notin \mathcal{I}_{jt}$. The shock ε_{jt} is assumed to be independent of the within period variation in the information set $\mathbb{P}(\varepsilon_{jt}|\mathcal{I}_{jt}) = \mathbb{P}(\varepsilon_{jt})$. Without loss of generality, we normalize the mean of ε_{jt} to be zero.

4.3 Revenue Production Function

We only have data on the revenue for firms at the line of business level and not the amount of output produced. Thus, we build the revenue production function by assuming a CES demand function of the form

$$\frac{P_{jt}}{P_t} = \left(\frac{Y_{jt}}{Y_t}\right)^{\frac{1}{\eta}} e^{\chi_{jt}}, \tag{5}$$

[16] In Argente et al. (2020), one component of productivity is a CES function of private and public inputs, and production is linear in labor input.

where P_{jt} is the output price of product j, P_t is the industry price index, Y_t is the quantity index that plays the role of an aggregate demand shifter, $\eta > 0$ represents the elasticity of demand, and χ_{jt} is a demand shock that is observed by the firm (Klette and Griliches, 1996; Grieco et al., 2016; Gandhi et al., 2020). The revenue production function is then

$$R_{jt} = \Lambda_t A_{jt}^{\zeta} F_{jt}^{\zeta} e^{\chi_{jt}}, \tag{6}$$

where $R_{jt} \equiv P_{jt}Y_{jt}$ is the annual revenue from product j in year t, $\zeta \equiv \frac{1}{\eta} + 1$, and $\Lambda_t \equiv P_t Y_t^{1-\zeta}$. Plugging in the functional form for F_{jt} and taking log gives

$$r_{jt} = \frac{\zeta\gamma}{\rho} \log\left(\alpha \tilde{H}_{jt}^{\rho} + (1-\alpha)\tilde{C}_{jt}^{\rho}\right) + \lambda_t + \zeta\varepsilon_{jt} + \nu_{jt}.$$

Here, $\zeta\gamma$ represents the returns to scale of the revenue production function.

5 Identification and Estimation

Although our firms produce across many different lines of business, we can apply econometric techniques for estimating single-output production functions since we observe input allocations at the product level. While we cannot identify the full set of production function parameters without output price data (Klette and Griliches, 1996; De Loecker, 2011; Kirov et al., 2023; Kasahara and Sugita, 2020), we can identify enough parameters to study the economies of scope, including the elasticity between private and public inputs and the returns to scale to the revenue production function.

Following Gandhi et al. (2020), we identify the production function parameters based on the moment conditions derived from two sets of assumptions. First, we use the firms' static profit maximization condition with respect to flexible inputs to derive moments from input share equations.[17] Second, we impose a Markov assumption on the stochastic process of the unobserved productivity shock to add dynamic panel moments. Our econometric approach allows us to identify revenue production function parameters based on standard identification assumptions while abstracting from the complex dynamic optimization problem of choosing shared inputs in the multi-output

[17] Also see Grieco et al. (2016) for a similar procedure but with parametric production functions.

production problem.[18]

5.1 Input Share Moments

We assume material and labor are flexible inputs. To choose the optimal level of flexible inputs given its information set, the producer of product j solves a static profit maximization problem

$$\max_{X_{jt}} \mathbb{E}[P_{jt}Y_{jt}|\mathcal{I}_{jt}] - X_{jt}, \tag{7}$$

where X_{jt} is the expenditure on the flexible input.[19] Rearranging the first-order conditions with respect to X_{jt} gives the input share equations

$$s_{jt}^X = \log \zeta + \log \xi_{jt}^X + \log \mathbb{E}[e^{\zeta \varepsilon_{jt}}] - \zeta \varepsilon_{jt},$$

where $s_{jt}^X \equiv \log \frac{X_{jt}}{R_{jt}}$ is the log of expenditure on input X relative to revenue, and $\xi_{jt}^X \equiv \frac{\partial Y_{jt}}{\partial X_{jt}} \frac{X_{jt}}{Y_{jt}}$ is the output elasticity with respect to input X.[20]

Following Grieco et al. (2016), we use normalized variables $\tilde{X}_{jt} = X_{jt}/\bar{X}$, where $\bar{X}$ represents the geometric mean of X_{jt} across the sample.[21] The nested CES functional form (3) then yields the elasticity with respect to flexible private input $X_{jt} \in \{M_{jt}, L_{jt}\}$ as

$$\xi_{jt}^X = \beta_X \gamma \frac{\alpha \tilde{H}_{jt}^\rho}{\alpha \tilde{H}_{jt}^\rho + (1-\alpha)\tilde{C}_{jt}^\rho}.$$

The assumption on unexpected productivity shock ε_{jt} ensures

$$\mathbb{E}[\varepsilon_{jt}|\mathcal{I}_{jt}] = 0, \tag{8}$$

so variables that are functions of period t information set $\mathcal{I}_{jt}$ serve as valid instruments. Note that estimates of $\mathbb{E}[e^{\zeta \varepsilon_{jt}}]$ can be obtained using residuals from a preliminary estimation.[22]

[18] In practice, the decision to employ common input across production lines may hinge on technological compatibility with product characteristics, significant fixed costs for setup, and the role of centralized management in reducing operational complexity.

[19] We express the profit maximization problem in terms of expenditure since we observe inputs in dollar units.

[20] See Appendix A.1 for the derivation of the input share equations.

[21] See Klump et al. (2012) and Grieco et al. (2016) for more information on the importance of normalizing inputs when estimating CES production functions.

[22] For example, one may obtain preliminary estimates of production function parameters after assuming $\varepsilon_{jt} = 0$

5.2 Dynamic Panel Moments

Taking the log of the revenue production function (6) gives

$$r_{jt} = \zeta f_{jt} + \lambda_t + \zeta\varepsilon_{jt} + \nu_{jt}, \tag{9}$$

where $\nu_{jt} \equiv \zeta\omega_{jt} + \chi_{jt}$. We assume that ν_{jt} follows a linear, first order Markov stochastic process

$$\nu_{jt} = \mu_0 + \mu_1\nu_{jt-1} + \eta_{jt}, \tag{10}$$

where the error term η_{jt} satisfies $\mathbb{E}[\eta_{jt}|\mathcal{I}_{jt-1}] = 0$.[23] Plugging in $\nu_{jt} = r_{jt} - \lambda_t - \zeta f_{jt} - \zeta\varepsilon_{jt}$ and rearranging gives

$$r_{jt} = \zeta f_{jt} + \mu_1(r_{jt-1} - \zeta f_{jt-1}) + \phi_t + \eta^*_{jt}, \tag{11}$$

where $\phi_t = \mu_0 + \lambda_t - \mu_1\lambda_{t-1}$, and $\eta^*_{jt} = \eta_{jt} + \zeta\varepsilon_{jt} - \mu_1\zeta\varepsilon_{jt-1}$. To estimate the production function parameters via the dynamic panel equation (11), we assume

$$\mathbb{E}[\eta^*_{jt}|\mathcal{I}_{jt-1}, \phi_t] = 0, \tag{12}$$

which would be satisfied if the aggregate prices and outputs are realized independently from product j-specific variables.[24] Then, together with time-fixed effects, any variables that are functions of $\mathcal{I}_{jt-1}$ serve as valid instruments.

almost surely (so that $\log \mathbb{E}[e^{\zeta\varepsilon_{jt}}] = 0$) and use the corresponding residuals to calculate $\mathbb{E}[\widehat{e^{\zeta\varepsilon_{jt}}}]$. We obtain estimate of $\mathbb{E}[e^{\zeta\varepsilon_{jt}}]$ by running a flexible second-order polynomial regression of input shares to private and common inputs.

[23] More specifically, assume that ν_{jt} follows a first-order Markov process so that $\mathbb{P}(\nu_{jt}|\mathcal{I}_{jt-1}) = \mathbb{P}(\nu_{jt}|\nu_{jt-1})$. Then, we can write the stochastic process of ν_{jt} as $\nu_{jt} = h(\nu_{jt-1}) + \eta_{jt}$, where $h(\nu_{jt-1}) = \mathbb{E}[\nu_{jt}|\nu_{jt-1}]$ and η_{jt} is the innovation tern that is unanticipated at period $t-1$ and satisfies $\mathbb{E}[\eta_{jt}|\mathcal{I}_{jt-1}] = 0$. Imposing a linear function form $h(\nu_{jt-1}) = \mu_0 + \mu_1\nu_{jt-1}$ gives (10).

[24] If $\phi_t \perp \eta^*_{jt}|\mathcal{I}_{jt-1}$, $\mathbb{E}[\eta^*_{jt}|\mathcal{I}_{jt-1}, \phi_t] = \mathbb{E}[\eta^*_{jt}|\mathcal{I}_{jt-1}] = 0$, where the last equality follows from our assumptions that ensure $\mathbb{E}[(\eta_{jt}, \varepsilon_{jt}, \varepsilon_{jt-1})|\mathcal{I}_{jt-1}] = 0$.

5.3 Combined Moments

Given the nested CES form of the production function, the input share equation and the dynamic panel equation become

$$s_{jt}^X = \log\psi + \log\beta_X + \log\left(\frac{\alpha\tilde{H}_{jt}^\rho}{\alpha\tilde{H}_{jt}^\rho + (1-\alpha)\tilde{C}_{jt}^\rho}\right) + \log\mathbb{E}[e^{\zeta\varepsilon_{jt}^X}] - \zeta\varepsilon_{jt}^X, \tag{13}$$

$$r_{jt} = \frac{\psi}{\rho}\log\left(\alpha\tilde{H}_{jt}^\rho + (1-\alpha)\tilde{C}_{jt}^\rho\right) + \mu_1\left(\log r_{jt-1} - \frac{\psi}{\rho}\log\left(\alpha\tilde{H}_{jt-1}^\rho + (1-\alpha)\tilde{C}_{jt-1}^\rho\right)\right) + \phi_t + \eta_{jt}^*, \tag{14}$$

where $\psi \equiv \zeta\gamma$ is the returns-to-scale parameter on the revenue production function. The identifiable parameters are $(\beta_m, \beta_l, \beta_k, \beta_e, \psi, \mu_1, \alpha, \delta, \rho)$ and time fixed-effects. To identify the production function parameters using the generalized method of moments approach, we form moment equations as

$$\mathbb{E}[\zeta\varepsilon_{jt}^X\tilde{Z}_{jt}^1] = 0,$$
$$\mathbb{E}[\eta_{jt}^*\tilde{Z}_{jt}^2] = 0.$$

We require $\tilde{Z}_{jt}^1$ to include variables that are functions of $\mathcal{I}_{jt}$ and $\tilde{Z}_{jt}^2$ to include time fixed effects and those that are functions of $\mathcal{I}_{jt-1}$. In $\tilde{Z}_{jt}^1$, we include the log of (normalized) private and common inputs and their quadratic terms. In $\tilde{Z}_{jt}^2$, we include time fixed effects, the log of contemporaneous non-flexible private inputs and common inputs, lagged revenue, lagged private and common inputs, and their quadratic terms.[25]

5.4 Measurement of Variables

We measure revenue and inputs as follows. Revenue is total sales and transfers at the line-of-business level. Materials is the total cost of materials. Labor is the total payroll. Capital is the net traceable plant, property, and equipment. Management expenses are the sum of general, administrative, media advertising, and other selling expenses. For capital and management expenses, we measure the private and public inputs as the traceable and non-traceable parts of the input, respectively.

[25] Note that $R_{jt-1}/e^{\zeta\varepsilon_{jt-1}} \in \mathcal{I}_{jt-1}$, but $R_{jt-1} \notin \mathcal{I}_{jt-1}$ since ε_{jt-1} is unknown to the firm at period $t-1$. We use R_{jt-1} as a proxy for $R_{jt-1}/e^{\zeta\varepsilon_{jt-1}}$.

We deflate the values of all variables to 1977 dollars. For output and materials, we match shipment and materials deflators from the NBER Productivity Database (Bartelsman and Gray, 1996; Becker et al., 2021) using line of business to SIC 1977 and SIC 1977 to SIC 1987 concordances. For capital, we use a combined deflator of the investment deflator from the NBER Productivity Database with the ratio of the current cost to the historical cost of fixed assets, available from the Bureau of Economic Analysis (BEA) at the 2-digit SIC level (U.S. Bureau of Economic Analysis, 2025a,b). We deflate labor and management expenses using the consumer price index (CPI). Finally, we drop observations that have zero common inputs. Table 3 provides the summary statistics for each variable (in logs) entering the production function.

Table 3: Summary Statistics for Production Inputs and Outputs

	Line-of-Business Level					Firm Level				
	Count	Mean	SD	Min	Max	Count	Mean	SD	Min	Max
Private Inputs										
Revenue	9,856	10.95	1.32	3.49	17.16	1,193	13.46	1.11	8.34	17.44
Materials	9,856	10.13	1.43	1.18	16.99	1,193	12.80	1.20	7.83	17.18
Labor	9,856	9.20	1.34	0.06	15.60	1,193	12.48	1.35	6.77	16.07
Capital	9,856	9.66	1.68	0.44	15.90	1,193	12.48	1.35	6.77	16.07
Management	9,856	8.61	1.51	0.21	14.60	1,193	11.61	1.60	2.08	16.23
Common Inputs										
Capital	9,856	9.83	1.57	3.43	14.53	1,193	11.49	1.91	3.56	17.38
Management	9,856	9.94	1.22	1.39	14.03	1,193	11.61	1.60	2.08	16.23

Notes: The values represent the log of 1977 dollars at the firm-line of business level.

6 Results

Given our relatively small sample size, we first estimate (3) assuming the same production parameters across all lines of business. We then examine heterogeneity in these estimates across a variety of dimensions. Using our production function estimates, we measure revenue elasticities at the line of business and firm level. We can also estimate productivity at the line of business level and examine how much of productivity dispersion occurs within firms.

6.1 Production Function Estimates

Table 4 reports the estimated production function parameters. The first column reports the estimates from our baseline specification that assumes material and labor are flexible inputs, as the literature on production functions has often assumed (Raval, 2023).

Table 4: Production Function Estimates

	(1)	(2)	(3)
β_m	0.48	0.66	0.61
	(0.06)	(0.11)	(0.03)
β_l	0.21	0.17	0.27
	(0.02)	(0.07)	(0.01)
β_k	0.11	0.08	0.04
	(0.03)	(0.03)	(0.02)
β_e	0.20	0.09	0.08
	(0.06)	(0.05)	(0.04)
ψ	1.00	0.71	0.74
	(0.09)	(0.16)	(0.05)
μ_1	0.81	0.98	0.98
	(0.07)	(0.09)	(0.02)
α	0.95	0.97	0.98
	(0.01)	(0.01)	(0.01)
δ	0.24	0.25	0.41
	(0.17)	(0.13)	(0.26)
ρ	0.60	0.56	0.00
	(0.15)	(0.56)	-
σ	2.50	2.27	1.00
Obs.	6,524	6,524	6,524
Flexible	$\{M, L\}$	$\{M\}$	$\{M, L\}$
Function	CES	CES	CD

Notes: Nonparametric bootstrap standard errors (B = 499, resampled at the line-of-business level) are shown in parentheses.

Of the Cobb-Douglas parameters for the line-of-business-specific inputs, materials get the highest weight at 0.48, followed by labor at 0.21, management at 0.20, and capital at 0.11. For public inputs, 24% of the weight is on capital and 76% on management. The returns to scale on the revenue production function ψ is estimated to be unity, indicating the revenue production function exhibits constant returns to scale.[26]

Our estimate of the distribution parameter $\alpha = 0.95$ suggests that 5% of the weight in the CES function is on the common input, implying the production function exhibits economies of scope from shared inputs. Finally, we estimate $\rho > 0$, indicating that private and public inputs are

[26] The product of the demand elasticity parameter ζ $(= \frac{1}{\eta} + 1)$ and the economies of scale parameter γ is identified, but the two parameters cannot be separately identified without output price data.

substitutes; the implied elasticity of substitution is $\sigma = \frac{1}{1-\rho} = 2.50$. To our knowledge, we are the first researchers to estimate firms' elasticity of substitution between private and public inputs.[27]

We also estimate the production function parameters under an alternative assumption that assumes that only materials is flexible in the second column of Table 4. We find broadly consistent estimates with this specification, with an elasticity of substitution of 2.3 and 3% of the weight in the CES production function on the public input. However, the standard error for the elasticity of substitution parameter is significantly larger.

Finally, the third column of Table 4 reports an estimate from a Cobb-Douglas specification that imposes $\rho = 0$ (i.e., $\sigma = 1$) a priori, as assumed in Cairncross et al. (2024) and Khmelnitskaya et al. (2024). While we continue to estimate α as less than 1 at 0.98, we cannot reject the null hypothesis that $\alpha = 1$. In addition, we find lower estimates of returns to scale ($\psi = 0.74$) and a higher weight on capital for the public input ($\delta = 0.41$).

Heterogeneity

To account for potential differences in production function parameters across the firms and lines of business in our data, we examine how our estimates vary across three cuts of our sample. First, we consider durable and non-durable products separately.[28] Second, we compare firms with high scope and low scope, where high scope firms have 10 or more lines of business. Finally, we examine high and low size firms, where high size firms have higher than median firm-level revenues. For all of these specifications, we maintain the nested CES production function and assume that materials and labor are flexible inputs.

In Table 5, we report the production function estimates across these specifications.[29] Overall, we find limited heterogeneity in production function parameters, as estimates across these subsamples are quantitatively and qualitatively similar to our baseline estimates with the full sample. For example, we find α ranges around 0.93–0.98, which indicates that common inputs have small but

[27] The empirical literature has focused on the micro and macro elasticity between capital and labor, with most estimates of this elasticity below one. See, for example, Doraszelski and Jaumandreu (2018); Oberfield and Raval (2021); Raval (2019); Zhang (2019). Also see Raval (2017) and Knoblach et al. (2020) for multiple meta-analyses.

[28] We define non-durables as lines of business whose two-digit SIC code is 20, 21, 22, 23, 26, 27, 28, 29, 30, or 31. Durables have two-digit SIC code of 24, 25, 32, 33, 34, 35, 36, 37, 38, or 39.

[29] We report GMM standard errors that ignore the sampling error from the first-stage estimation of $\mathbb{E}[e^{\zeta \varepsilon_{jt}}]$ for simplicity and tractability. Although two-stage standard error correction will increase the standard errors, the additive dependence of the input share equations on $\log \mathbb{E}[e^{\zeta \varepsilon_{jt}}]$ reduces the likelihood of severe bias in the second-stage standard errors. We also focus on the point estimates of the parameters.

positive distribution weights in the production function, consistent with economies of scope from shared inputs. Moreover, common management expenses have higher weights than common capital, as δ consistently remains below half. We also find that private and shared inputs are substitutes (i.e. $\sigma > 1$) for all specifications except for the non-durables sample.

Table 5: Heterogeneity in Production Function Estimates

	(1) Durables	(2) Non-durables	(3) High Scope	(4) Low Scope	(5) High Size	(6) Low Size
β_m	0.54	0.55	0.55	0.47	0.63	0.51
	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)	(0.01)
β_l	0.27	0.17	0.22	0.22	0.23	0.25
	(0.01)	(0.00)	(0.00)	(0.00)	(0.01)	(0.00)
β_k	0.06	0.13	0.06	0.10	0.05	0.09
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
β_e	0.14	0.15	0.16	0.20	0.10	0.16
	(0.02)	(0.02)	(0.01)	(0.01)	(0.02)	(0.01)
ψ	0.88	0.93	0.93	1.01	0.76	0.97
	(0.02)	(0.02)	(0.02)	(0.01)	(0.02)	(0.01)
μ_1	0.96	0.90	0.91	0.80	0.97	0.83
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)
α	0.95	0.95	0.96	0.95	0.98	0.93
	(0.01)	(0.02)	(0.01)	(0.01)	(0.01)	(0.01)
δ	0.24	0.31	0.33	0.00	0.46	0.16
	(0.11)	(0.07)	(0.11)	(0.10)	(0.36)	(0.07)
ρ	0.48	-0.45	0.60	0.74	0.25	0.52
	(0.06)	(0.11)	(0.07)	(0.08)	(0.24)	(0.05)
σ	1.92	0.69	2.50	3.85	1.33	2.08
Obs.	3,589	2,935	3,298	3,226	3,274	3,250

Notes: GMM HAC standard errors are reported in parentheses. The standard errors ignore the sampling error from the first-stage estimation of $\log \mathbb{E}[e^{\zeta \varepsilon_{jt}}]$. Row σ reports the elasticity of substitution implied by the estimated ρ.

6.2 Revenue Elasticities

We now examine the revenue elasticities for public inputs. For public inputs, the revenue elasticity for a given line of business is different from the revenue elasticity for the business as a whole. The latter elasticity would be most relevant when a business has to decide on allocating resources to public inputs. We first derive product-level revenue elasticities and then show how to aggregate to firm-wide revenue elasticities.

Product-Level Revenue Elasticities

Given that the revenue production function under the nested CES demand assumption takes the form (6), the revenue elasticities with respect to inputs are

$$\frac{\partial \log R_{jt}}{\partial \log X_{jt}} = \begin{cases} \psi \beta_X \frac{\alpha \tilde{H}_{jt}^{\rho}}{\alpha \tilde{H}_{jt}^{\rho} + (1-\alpha)\tilde{C}_{jt}^{\rho}} & \text{if } X_{jt} \in \{M_{jt}, L_{jt}, K_{jt}, E_{jt}\}, \\ \psi \delta \frac{(1-\alpha)\tilde{C}_{jt}^{\rho}}{\alpha \tilde{H}_{jt} + (1-\alpha)\tilde{C}_{jt}^{\rho}} & \text{if } X_{jt} = \mathcal{K}_{jt}, \\ \psi (1-\delta) \frac{(1-\alpha)\tilde{C}_{jt}^{\rho}}{\alpha \tilde{H}_{jt} + (1-\alpha)\tilde{C}_{jt}^{\rho}} & \text{if } X_{jt} = \mathcal{E}_{jt}, \end{cases} \tag{15}$$

all of which are identified.[30]

We compare the distribution of the identified revenue elasticities across inputs in Table 6. Materials exhibit the highest revenue elasticities with a mean of 0.45. Revenue elasticities with respect to common inputs appear smaller than those for private inputs, with a mean elasticity of 0.02 for shareable capital and 0.05 for shareable management. Yet, as we show below, change in common inputs affect the revenue of all lines of business of the firm, so the effect of increasing a common input on total firm revenue will depend on the number of lines of business in a firm.

Table 6: Distribution of Revenue Elasticities with Respect to Inputs

Input	Count	Mean	SD	Min	Q1	Median	Q3	Max
Private Inputs								
Materials	9,856	0.45	0.03	0.09	0.44	0.46	0.47	0.48
Labor	9,856	0.20	0.01	0.04	0.19	0.20	0.21	0.21
Capital	9,856	0.10	0.01	0.02	0.10	0.11	0.11	0.11
Management	9,856	0.18	0.01	0.04	0.18	0.19	0.19	0.20
Common Inputs								
Capital	9,856	0.02	0.02	0.00	0.01	0.01	0.02	0.19
Management	9,856	0.05	0.05	0.00	0.02	0.03	0.06	0.62

Notes: Q1 and Q3 represent the first and third quartiles, respectively.

[30] The output elasticities with respect to inputs are identified up to a multiplicative constant ζ since the demand elasticity parameter and scale parameter are not separately identified. Since $\zeta > 1$, the reported distribution informs the upper bound of true output elasticities.

Aggregate Revenue Elasticities

We thus now examine the aggregate revenue elasticities for capital and management expenses, and indeed find higher elasticities at the aggregate level than the line of business level. As we show below, aggregate elasticities will depend on whether an increase in the overall input affects public or private inputs or both.

We derive the aggregate elasticities as follows. For a given firm and year, let $R = \sum_j R_j$ be the aggregate revenue summed across the firm's lines of business indexed by j. Let $X = \sum_j X_j + X_C$, where X_j is the product-specific input and X_C is the common input. Finally, assume that

$$\begin{aligned} dX_j &= \pi_j dX, \\ dX_C &= \pi_C dX, \\ \sum_j \pi_j + \pi_C &= 1, \end{aligned} \tag{16}$$

where the proportionality coefficients π_j and π_C characterize how much each of X_j and X_C increase to create an increase of aggregate input X by one unit. In Section A.2, we derive the aggregate elasticity as

$$\frac{\partial \log R}{\partial \log X} = \sum_j s_j^R \left(\left(\frac{\pi_j}{s_j^X} \right) \frac{\partial \log R_j}{\partial \log X_j} + \left(\frac{\pi_C}{s_C^X} \right) \frac{\partial \log R_j}{\partial \log X_C} \right), \tag{17}$$

where $s_j^R \equiv R_j/R$, $s_j^X = X_j/X$, and $s_C^X = X_C/X$. Equation (17) shows that the value of aggregate elasticity depends on which components of the input drive the change.

We consider three assumptions on which inputs change; that is, on the proportionality coefficients π_j and π_C. First, we assume private inputs and common input increase in proportion to their share (i.e., $\pi_j = s_j^X$ and $\pi_C = s_C^X$), so that the percentage change for each input is the same across private and public inputs. In that case, the aggregate elasticity has a particularly simple form as the revenue share weighted sum of the private and public elasticities:

$$\frac{\partial \log R}{\partial \log X} = \sum_j s_j^R \left(\frac{\partial \log R_j}{\partial \log X_j} + \frac{\partial \log R_j}{\partial \log X_C} \right). \tag{18}$$

In the second case, we assume that only the public input increases but private inputs do not (i.e., $\pi_j = 0$ and $\pi_C = 1$). In the last case, we assume that all private inputs increase in proportion to their share (so they all have the same percentage change in the input), but the common input does not increase (i.e., $\pi_j = s_j^X/(1-s_C^X)$ and $\pi_C = 0$).

Table 7: Distribution of Aggregate Revenue Elasticities with Respect to Inputs

Input	Count	Mean	SD	Min	Q1	Median	Q3	Max
Case 1: $\pi_j = s_j^X$, $\pi_C = s_C^X$								
Capital	1,193	0.11	0.00	0.11	0.11	0.11	0.12	0.14
Management	1,193	0.21	0.01	0.20	0.20	0.21	0.22	0.32
Case 2: $\pi_j = 0$, $\pi_C = 1$								
Capital	1,193	0.25	0.69	0.00	0.05	0.10	0.21	14.40
Management	1,193	0.15	0.21	0.02	0.07	0.11	0.18	4.64
Case 3: $\pi_j = s_j^X/(1-s_C^X)$, $\pi_C = 0$								
Capital	1,193	0.14	0.17	0.11	0.11	0.11	0.12	3.06
Management	1,193	0.38	0.94	0.19	0.21	0.23	0.28	16.59

Notes: Q1 and Q3 represent the first and third quartiles, respectively. Firm-level aggregate elasticities are obtained using equation (17) for the given assumption on the proportionality coefficients.

Table 7 reports the distribution of aggregate elasticities for each assumption on the distribution of the increase in aggregate input. We highlight two findings. First, we find substantially higher elasticities for the common input at the aggregate level. For example, the median aggregate revenue elasticities for capital and management from increasing the common input are 0.10 and 0.11, compared to 0.01 and 0.03 at the line of business level. Second, aggregate elasticities depend substantially on how the increase in an input is distributed across public and private inputs. The median aggregate elasticities are 0.11 and 0.23 for capital and management when the private input for each line of business increases in proportion to their shares, compared to 0.10 and 0.11 when only the public input increases. The literature typically estimates elasticities for inputs using firm-level production data; the magnitude of such elasticities will depend on whether changes in inputs observed in the data are from changes in public or private inputs.

6.3 Within-Firm Heterogeneity in Revenue Productivity

We document a high level of within-firm heterogeneity in revenue productivity. Recall that the revenue production function is given by

$$R_{jt} = \tilde{A}_{jt} \left(\alpha \tilde{H}_{jt}^{\rho} + (1-\alpha) \tilde{C}_{jt}^{\rho} \right)^{\frac{\psi}{\rho}}, \tag{19}$$

where $\tilde{A}_{jt} \equiv \Lambda_t A_{jt}^{\zeta} e^{\chi_{jt}}$. Using our estimates of the production function parameters, we can recover the revenue productivity term $\tilde{A}_{jt}$ from (19).[31]

First, for each firm f with multiple lines of business, we compute $\log(\tilde{A}_{f,t}^{max}/\tilde{A}_{f}^{min})$, where $\tilde{A}_{ft}^{max}$ and $\tilde{A}_{ft}^{min}$ represent the maximum and minimum of $\tilde{A}_{jt}$ across all products j being produced by firm f. Figure 3 plots the distribution of $\log(\tilde{A}_{f,t}^{max}/\tilde{A}_{f,t}^{min})$ using the observations from the last period. We observe significant heterogeneity in TFPR across lines of business within firms, with the median of 0.58, which translates to the median firm having maximum TFPR that is approximately $\exp(0.58) \approx 1.78$ times larger than the minimum TFPR.[32]

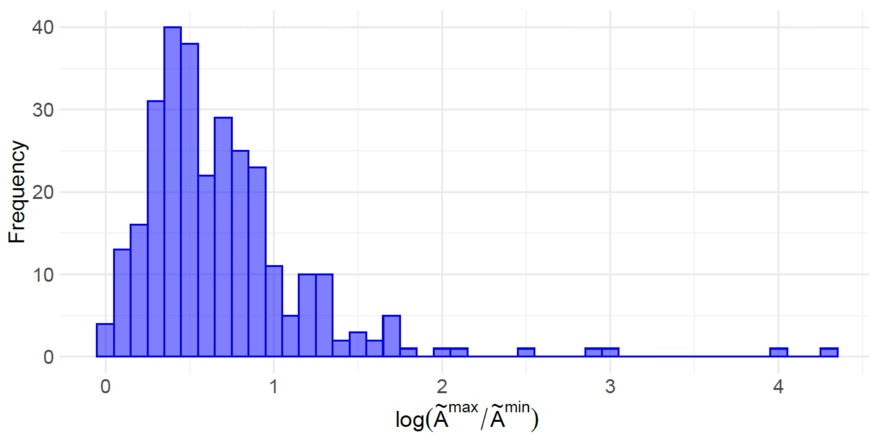

Figure 3: Within-Firm Heterogeneity in Revenue Productivity

Next, we follow Orr (2022) and decompose the variance of productivity into within firm and between firm productivity differences. Letting $\tilde{a}_j = \log \tilde{A}_j$, we have $\text{Var}(\tilde{a}_j) = \text{Var}_{\text{within}}(\tilde{a}_j) + \text{Var}_{\text{across}}(\tilde{a}_j)$, where $\text{Var}(\tilde{a}_j)$ is the total variance of $\tilde{a}_j$, $\text{Var}_{\text{within}}(\tilde{a}_j)$ is the within-firm variance, and $\text{Var}_{\text{across}}(\tilde{a}_j)$ is the cross-firm variance.[33] In Table 8, we report the result of the variance decomposition. We find that 75.5% of the variance is due to within-firm heterogeneity in the productivity terms. Our estimate of within-firm heterogeneity is larger than that found in Orr (2022), who examine within-plant vs. between-plant productivity differences for Indian manufacturing plants.

[31] A caveat in the interpretation of our results is that the within-firm heterogeneity in revenue productivity cannot necessarily be interpreted as signs of within-firm heterogeneity in output productivity because the $\tilde{A}_{jt}$ includes product-specific demand shocks. Our data allow us to measure TFPR (revenue-based total factor productivity) and not TFPQ (quantity-based total factor productivity).

[32] The mean, 1Q, median, and 3Q of $\log(\tilde{A}_{f,t}^{max}/\tilde{A}_{f}^{min})$ across firms are 0.69, 0.37, 0.58, and 0.87, respectively.

[33] Letting $\tilde{a}_j = (\frac{1}{J_f}\sum_{j\in f}\tilde{a}_j) + (\tilde{a}_j - \frac{1}{J_f}\sum_{j\in f}\tilde{a}_j)$, the variance of the first term is the cross-firm variance, and the variance of the second term is the within-firm variance. The two terms are uncorrelated by construction, so the total variance is the sum of the variances of the two terms.

Table 8: Revenue Productivity Variance Decomposition

	Across	Within	Total
Variance	0.024	0.074	0.098
Percentage	0.245	0.755	1.000

Notes: Table reports the result of variance decomposition using the 1977 sample. Total variance is decomposed as the sum of cross-firm variance and within-firm variance.

7 Counterfactual Analysis

7.1 Revenue Loss from Reduction in Common Input

We assess the degree of economies of scope by calculating the counterfactual loss in revenue following a reduction in common input. Let R^*_{jt} be the counterfactual revenue associated with a ceteris paribus change in common input to $C^*_{jt} = (1-\phi)C_{jt}$, where $\phi \in [0,1]$. Given the revenue function (6), the change in revenue when reducing C_{jt} by a fraction ϕ is $\%\Delta R_{jt} = R^*_{jt}/R_{jt} - 1$, or

$$\%\Delta R_{jt} = \left(\frac{\alpha \tilde{H}^{\rho}_{jt} + (1-\alpha)\tilde{C}^{*\rho}_{jt}}{\alpha \tilde{H}^{\rho}_{jt} + (1-\alpha)\tilde{C}^{\rho}_{jt}} \right)^{\psi/\rho} - 1. \tag{20}$$

We can identify (20) using the production function parameter estimates and observed line-of-business-level input data.

Reductions in the common input can lead to substantial revenue losses for firms. Table 9 reports the expected revenue loss by the number of lines of business for different assumptions on the reduction parameter ϕ as well as average across firms. For the average firm, the expected loss in revenue is 0.7% for a 10% reduction in the public input, 1.7% for a 25% reduction, 3.6% for a 50% reduction, 6.0% for a 75% reduction, and 7.9% for a 90% reduction.

As one would expect from economies of scope, the decline in revenue is larger for firms with greater scope i.e., more lines of business. For example, when common input is reduced by 50%, we estimate an average decline of 1.9% for firms with one line of business, 2.5% for firms with two to three lines of business, 2.9% for firms with four to six lines of business, 3.1% for firms with seven to nine lines of business, and 4.2% for firms with greater than ten lines of business. The positive correlation between the number of lines of business and the expected revenue loss may be

Table 9: Expected Revenue Loss by the Number of Lines of Business

	Reduction in Common Input				
LOB	10%	25%	50%	75%	90%
1	0.4%	0.9%	1.9%	3.2%	4.2%
2 – 3	0.5%	1.2%	2.5%	4.1%	5.4%
4 – 6	0.5%	1.3%	2.9%	4.7%	6.2%
7 – 9	0.6%	1.5%	3.1%	5.2%	6.8%
10+	0.8%	2.0%	4.2%	6.9%	9.1%
All	0.7%	1.7%	3.6%	6.0%	7.9%

Notes: Table reports the expected revenue loss following a reduction in common inputs. Row "All" reports the unconditional average revenue loss across all firms and lines of business.

attributable to the fact that firms with more lines of business tend to have a higher level of common input.

7.2 Merger Synergies

A merger of two firms with non-overlapping production lines generates merger synergies from economies of scope because the stock of common input increases. We simulate mergers of firms with no overlap in lines of businesses to quantify the extent to which merger synergies from economies from scope can increase total revenues of the merging firms.

Suppose that firms A and B with no overlap in production lines merge. We assume the post-merger revenue from pooling common inputs is

$$R_{jt}^{\text{post}} = \tilde{A}_{jt}^{\text{pre}} \left(\alpha(\tilde{H}_{jt}^{\text{pre}})^{\rho} + (1-\alpha)(\tilde{C}_{jt}^{\text{post}})^{\rho} \right)^{\psi/\rho}. \tag{21}$$

Post-merger revenue in (21) assumes that the revenue productivity and product-specific private inputs are fixed, but common inputs are aggregated to $\tilde{C}_{jt}^{\text{post}} = (\tilde{\mathcal{K}}_{jt}^{\text{post}})^{\delta}(\tilde{\mathcal{E}}_{jt}^{\text{post}})^{1-\delta}$. We model the common input aggregation in two ways. First, we consider $\tilde{\mathcal{X}}_{jt}^{\text{post}} = \max\{\tilde{\mathcal{X}}_{jt}^{A}, \tilde{\mathcal{X}}_{jt}^{B}\}$ for $\tilde{\mathcal{X}}_{jt} \in \{\tilde{\mathcal{K}}_{jt}, \tilde{\mathcal{E}}_{jt}\}$. Such assumption would be valid if the firm adopts the best of two alternatives and pooling of common resources is impossible due to technological reasons.[34] Second, we consider

[34]For example, the merging firms are likely to adopt the best of two alternative employee training programs without needing to maintain both. Using maximum rather than sum also produces conservative estimates of merger synergies.

$\tilde{\mathcal{X}}_{jt}^{\text{post}} = \tilde{\mathcal{X}}_{jt}^{A} + \tilde{\mathcal{X}}_{jt}^{B}$. Such assumption would be valid if the firms' common inputs are perfect substitutes to each other.

We use the 1977 cross-section to simulate two-firm mergers. We consider 33,018 pairwise mergers of firms with no overlap in production lines.[35] Our simulation exercise asks how the merging firms' total revenues would change if they could pool their resources but is not an equilibrium exercise that allows firms to re-optimize.

Table 10 reports the distribution of predicted percentage changes in total revenue by common input aggregation type. We find that a merger of firms with no overlap in production lines increases total revenue by 1.6–2.6% on average and 1.0–2.0% at the median. Our results indicate that merging firms may boost their revenues by pooling shared inputs due to economies of scope.

Table 10: Predicted Percentage Change in Total Revenue from Mergers

Common Input Aggregation	Count	Mean	SD	Min	Q1	Median	Q3	Max
$\mathcal{X}^{\text{post}} = \max\{\mathcal{X}^A, \mathcal{X}^B\}$	33,018	1.6%	2.2%	0.0%	0.5%	1.0%	1.8%	40.2%
$\mathcal{X}^{\text{post}} = \mathcal{X}^A + \mathcal{X}^B$	33,018	2.6%	2.3%	0.1%	1.3%	2.0%	3.0%	40.6%

Notes: Table reports the distribution of percentage change in total revenue of the merging firms for 33,018 mergers of firms with no overlap in lines of businesses.

8 Conclusion

In this article, we have examined the degree of economies of scope using data on large manufacturing firms from the FTC's Line of Business surveys. With the Line of Business data, we could examine inputs at the line-of-business level as well as information on the degree of shared or common inputs to the firm as a whole. We found that firms report substantial amounts of shared inputs, and that the ratio of the shared input to private input was positively associated with firm size and scope.

These facts motivated a nested CES model of production that included common inputs. We estimated this model using the line-of-business data and found that the common input had a positive output elasticity and was substitutable with line-of-business-specific inputs. After estimating revenue productivity, we find three-quarters of productivity differences are within firms rather than across firms. Finally, we found considerable declines in revenue from reducing the amount of com-

[35] Given 305 firms with positive common inputs in 1977, considering all pairs of firms gives $\binom{305}{2} = 46{,}360$ merger simulations. After limiting the scope to only those with no overlap in lines of business, we have 33,018 mergers or approximately 71.2% of all possible mergers.

mon input used in production, and modest merger synergies in simulations from combining the shared inputs in the merged firm.

We see several avenues for future research. First, our econometric strategy was agnostic about how much common input firms employ. Future research could examine how firms choose such inputs, such as what triggers firms to start incorporating common inputs in their production lines and how firms adjust common inputs in response to demand shocks. Second, researchers could examine how mergers and other changes in the structure of the firm affect shared inputs and economies of scope.

More broadly, we examined economies of scope from shared inputs in manufacturing in the 1970s. Retail trade has seen a massive increase in the importance of national firms (Hsieh and Rossi-Hansberg, 2023), which may indicate large economies of scope from operating in different geographic and product markets. The source of economies of scope for retail trade or the digital economy could be quite different than for manufacturing.

A Technical Appendix

A.1 Derivation of Input Share Equations

Suppose that $Y_{jt} = A_{jt}F_{jt}$ and that the static profit maximization problem is given as (7). Rewriting the objective function using the CES demand function (5) and the i.i.d. assumption on ε_{jt} gives

$$\Lambda_t Y_{jt}^{\zeta} \frac{\mathbb{E}[e^{\zeta\varepsilon_{jt}}]}{e^{\zeta\varepsilon_{jt}}} e^{\chi_{jt}} - X_{jt}.$$

The first-order condition with respect to X_{jt} gives

$$\Lambda_t \zeta Y_{jt}^{\zeta-1} A_{jt} \frac{\partial F_{jt}}{\partial X_{jt}} \frac{\mathbb{E}[e^{\zeta\varepsilon_{jt}}]}{e^{\zeta\varepsilon_{jt}}} e^{\chi_{jt}} = 1.$$

Rearranging the above using (6) gives

$$\underbrace{\Lambda_t Y_{jt}^{\zeta} e^{\chi_{jt}}}_{=R_{jt}} \underbrace{\left(\frac{\partial F_{jt}}{\partial X_{jt}} \frac{X_{jt}}{F_{jt}}\right)}_{=\xi_{jt}^X} \underbrace{\frac{A_{jt}F_{jt}}{Y_{jt}}}_{=1} \frac{1}{X_{jt}} \frac{\mathbb{E}[e^{\zeta\varepsilon_{jt}}]}{e^{\zeta\varepsilon_{jt}}} = 1,$$

which in turn produces the input share equation

$$S_{jt}^X = \zeta \xi_{jt}^X \frac{\mathbb{E}[e^{\zeta\varepsilon_{jt}}]}{e^{\zeta\varepsilon_{jt}}},$$

where $S_{jt}^X = \frac{X_{jt}}{R_{jt}}$ is the input expenditure share relative to revenue, and $\xi_{jt}^X \equiv \frac{\partial F_{jt}}{\partial X_{jt}} \frac{X_{jt}}{F_{jt}}$ is the output elasticity with respect to input X. By taking log, we obtain

$$s_{jt}^X = \log \zeta + \log \xi_{jt}^X + \log \mathbb{E}[e^{\zeta\varepsilon_{jt}}] - \zeta\varepsilon_{jt}, \quad X \in \{M, L\},$$

where $s_{jt}^X \equiv \log S_{jt}^X$. The above equations represent the share equations from firms' static profit maximization conditions.

A.2 Derivation of Aggregate Revenue Elasticity

We derive expressions for aggregate revenue elasticities. Let $R = \sum_j R_j$ be the aggregate revenue of a given firm, summed across products indexed by j. Let $X = \sum_j X_j + X_C$ be the aggregate input, where X_j is product-specific input and X_C is common input. We want to characterize the elasticity of R with respect to X (i.e., $\partial \log R / \partial \log X$), recognizing that the aggregate revenue elasticity with respect to aggregate input may be different depending on which component of X drives the change.

Let

$$\begin{aligned} dX_j &= \pi_j dX, \\ dX_C &= \pi_C dX, \\ \sum_j \pi_j + \pi_C &= 1, \end{aligned}$$

where the proportionality coefficients π_j and π_C characterize how much each of X_j and X_C increase to create an increase of X by one unit. We have

$$\begin{aligned} \frac{\partial R}{\partial X} &= \sum_j \frac{\partial R_j}{\partial X} \\ &= \sum_j \left(\sum_l \frac{\partial R_j}{\partial X_l} \frac{dX_l}{dX} + \frac{\partial R_j}{\partial X_C} \frac{dX_C}{dX} \right) \\ &= \sum_j \left(\frac{\partial R_j}{\partial X_j} \frac{dX_j}{dX} + \frac{\partial R_j}{\partial X_C} \frac{dX_C}{dX} \right) \\ &= \sum_j \left(\frac{\partial R_j}{\partial X_j} \pi_j + \frac{\partial R_j}{\partial X_C} \pi_C \right). \end{aligned}$$

Then

$$\begin{aligned} \frac{\partial \log R}{\partial \log X} &= \frac{\partial R}{\partial X} \frac{X}{R} \\ &= \frac{X}{R} \sum_j \left(\frac{\partial R_j}{\partial X_j} \pi_j + \frac{\partial R_j}{\partial X_C} \pi_C \right) \\ &= \frac{X}{R} \sum_j \left(\frac{\partial R_j}{\partial X_j} \frac{X_j}{R_j} \frac{R_j}{X_j} \pi_j + \frac{\partial R_j}{\partial X_C} \frac{X_C}{R_j} \frac{R_j}{X_C} \pi_C \right) \\ &= \sum_j \left(\frac{R_j}{R} \right) \left(\frac{\partial \log R_j}{\partial \log X_j} \left(\frac{X}{X_j} \right) \pi_j + \frac{\partial \log R_j}{\partial \log X_C} \left(\frac{X}{X_C} \right) \pi_C \right). \end{aligned}$$

Thus, the aggregate revenue elasticity with respect to aggregate input is

$$\frac{\partial \log R}{\partial \log X} = \sum_j s_j^R \left(\left(\frac{\pi_j}{s_j^X} \right) \frac{\partial \log R_j}{\partial \log X_j} + \left(\frac{\pi_C}{s_C^X} \right) \frac{\partial \log R_j}{\partial \log X_C} \right),$$

where $s_j^R \equiv R_j/R$, $s_j^X \equiv X_j/X$, and $s_C^X = X_C/X$.

- Case 1: If $\pi_j = s_j^X$ and $\pi_C = s_C^X$, then

$$\frac{\partial \log R}{\partial \log X} = \sum_j s_j^R \left(\frac{\partial \log R_j}{\partial \log X_j} + \frac{\partial \log R_j}{\partial \log X_C} \right).$$

- Case 2: If $\pi_j = 0$ and $\pi_C = 1$, then

$$\frac{\partial \log R}{\partial \log X} = \sum_j \frac{s_j^R}{s_C^X} \frac{\partial \log R_j}{\partial \log X_C}.$$

- Case 3: If $\pi_j = \frac{s_j^X}{1 - s_C^X}$ and $\pi_C = 0$, then

$$\frac{\partial \log R}{\partial \log X} = \sum_j \frac{s_j^R}{1 - s_C^X} \frac{\partial \log R_j}{\partial \log X_j}.$$

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