Web Appendix for Micro Data and Macro Technology*

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*Any opinions and conclusions expressed herein are those of the authors and do not necessarily represent the views of the U.S. Census Bureau or the Federal Trade Commission and its Commissioners. All results have been reviewed to ensure that no confidential information is disclosed.

B Data

This section discusses the construction of the data.

B.1 Economic Censuses

We use the 1987 through 2007 Census of Manufactures to estimate plant elasticities of substitution and demand. We remove all Administrative Record plants because these plants do not have data on output or capital. We also eliminate a set of outliers and missing values from the dataset. We first remove all plants born in the given Census year, as well as a small set of plants with missing age data. We then remove plants with zero, missing, or negative data for the equipment capital stock, structures capital stock, labor costs, value added, or materials. We also remove plants above the 99.5th percentile or below the 0.5th percentile of their 2-digit SIC industry on these variables to remove plants with potential data problems. Finally, we drop plants in Alaska and Hawaii as we do not have amenity instruments for these locations.

For capital costs, we multiply capital stock measures by rental rates of capital. For the capital stock, we use the Census constructed measure of perpetual inventory capital stock, which is constructed for structures and equipment capital separately. The Census uses book values of capital together with investment histories to construct these capital stocks; thus, they will be primarily based upon book values for plants that exist only in Census years, while for large plants always sampled in the ASM, they may be based on a long time span of continuous investment histories.

The Annual Survey of Manufactures tracks about 50,000 plants over five year panel rotations that are more heavily weighted towards large plants. We use the ASM to calculate the heterogeneity indices and materials shares. The ASM has data on plant investment over time as well as book values of the stock of capital, which are used by the Census to construct perpetual inventory measures of capital stocks. The ASM plant samples also have data on the value of non-monetary compensation given to employees, such as health care or retirement benefits, which we use to better measure payments to labor. We include non-monetary compensation as part of labor costs when we use the 2002 and 2007 Census of Manufactures, as these years include non-monetary labor compensation for all plants.

B.2 Rental Rates

We define the rental rate using the external real rate of return specification of Harper et al. (1989). The rental rate for industry n is defined as:

$$R_{i,t} = T_{i,t}(p_{i,t-1}r_{i,t} + \delta_{i,t}p_{i,t})$$

where $r_{i,t}$ is a constant external real rate of return of 3.5 percent, $p_{i,t}$ is the price index for capital in that industry, $\delta_{i,t}$ is the depreciation rate for that industry, and $T_{i,t}$ is the effective rate of capital taxation. We calculate $T_{i,t}$ following Harper et al. (1989) as:

$$T_{i,t} = \frac{1 - u_t z_{i,t} - k_{i,t}}{1 - u_t}$$

where $z_{i,t}$ is the present value of depreciation deductions for tax purposes on a dollar's investment in capital type i over the lifetime of the investment, $k_{i,t}$ is the effective rate of the investment tax credit, and u_{it} is the effective corporate income tax rate. We obtained $z_{i,t}$, u_{it}, and $k_{i,t}$ from Dale

Jorgenson at the asset year level; we then used a set of capital flow tables at the asset-industry level to convert these to the industry level.

To calculate depreciation rates $\delta_{i,t}$, we take depreciation rates from NIPA at the asset level and use the capital flow tables to convert them to the industry level. Our primary source of prices of capital $p_{i,t}$ are from NIPA, which calculates separate price indices for structures and equipment capital.

The capital flow tables and investment price series depend upon the industry definition; because the US switches from SIC basis to NAICS basis during this period, we construct separate rental price series for SIC 2 digit industries and NAICS 3 digit industries. Finally, when we examine the aggregate we have to aggregate all of the rental price series; we do so by calculating Tornqvist indices between equipment and structures capital for each industry, and then a Tornqvist index across rental rates for each industry. The Tornqvist indices allow for the share of equipment capital in industry capital and for the share of different industries in manufacturing capital to change over this period.

B.3 Local Wages

We construct measures of the local wage in order to estimate the elasticity of substitution across plants, using both worker and establishment level data to measure the local area wage. The primary dataset that we use is the Census 5 percent samples of Americans, together with the American Community Surveys. Both of those datasets have data on wages and local area geographic location for a large sample of workers.

To obtain the local wage, we first calculate the individual wage for workers with age between 20 and 65 who are employed in the private sector as workers earning a wage or salary and who do not live in group quarters. We calculate the wage as an hourly wage, defined as total yearly wage and salary income divided by total hours worked. We use the CPI to deflate wage income, which affects the wages matched to the 2007 Census of Manufactures, as these rely on information on workers over 5 different years of the ACS.

We measure total hours worked as weeks worked per year multiplied by hours worked per week. We remove all individuals with zero or missing income or zero total hours worked. In 2008 and 2009, only the intervalled number of weeks worked is available. We thus impute the number of weeks worked for individuals in 2008 and 2009 based on averages of the number of weeks worked from 2005 to 2007 from cells of the intervalled weeks worked, an indicator if the worker is female, an indicator if the worker is black, the educational attainment of the worker (as constructed below), and age (as a set of dummy variables for age intervals).

Total wage and salary income in the Population Censuses and American Community Surveys are topcoded. The topcode threshold is \$140,000 in 1990, \$175,000 in 2000, and the 99.5% of the state distribution of income for that year in the ACS years. For all cases, we impute the total wage and salary income to 1.5 times the topcode if the wage and salary income is topcoded, in line with Acemoglu and Angrist (2000).

Before calculating local area wages, we adjust measures of local wages for differences in worker characteristics through regressions with the individual log wage as a dependent variable. We include education through a set of dummy variables based upon the worker's maximum educational attainment, which include four categories: college, some college, high school degree, and high school dropouts. We define experience as the individual's age minus an initial age of working that depends upon their education status, and include a quartic in experience in the regression. We also have data on the race of workers and so include three race categories of white, black, and other, as well as an indicator for Hispanic origin and gender. We include six occupational categories: Managerial and Professional; Technical, Sales, and Administrative; Service, Farming, Forestry, and Fishing; Precision Production, Craft, and Repairers; and Operatives and Laborers. Finally, we include thirteen industrial categories: Agriculture, Forestry, and Fisheries; Mining; Construction; Manufacturing; Transportation, Communications and Other Public Utilities; Wholesale Trade; Retail Trade; Finance, Insurance, and Real Estate; Business and Retail Services; Personal Services; Entertainment and Recreation Services; Professional and Related Services; and Public Administration.

We then regress the local wage on all of these characteristics, with separate regressions by year. For wages matched to the 2007 Census of Manufactures which use multiple ACS years, we include year effects as well to allow the overall wage distribution to vary over time.

We then aggregate the residuals from this regression to the commuting zone level. The Population Census and ACS data only contain information on the Public Use Micro Area (PUMA) of the individual worker. Thus, we use crosswalks from Autor and Dorn (2013) in order to aggregate from the PUMA to the Commuting Zone. Since some PUMAs contain multiple commuting zones, we weight each residual wage by the multiple of the person weight in the Census or ACS and a weight that indicates the fraction of the PUMA in the given Commuting Zone. We then construct average residual wages for each commuting zone.

Because the Economic Census is conducted in different years from the Population Censuses, we match the 1987 and 1992 Censuses of Manufactures to wages from the 1990 Population Census, the 1997 and 2002 Censuses of Manufactures to wages from the 2000 Population Census, and the 2007 Census of Manufactures to the 2005-2009 American Community Surveys.

The second dataset that we use for our IV and panel data specifications is the Longitudinal Business Database, which contains data on payroll and employment for all US establishments. We construct the establishment wage as total payroll divided by total employment. We measure the local wage as the mean log wage at the commuting zone level, after regressing the log establishment wage on indicator variables for 4 digit SIC or 6 digit NAICS industry codes to remove industry effects. We match the Longitudinal Business Database to its equivalent year in the Census of Manufactures.

B.4 Instruments

We use three different sets of instruments for the local wage in order to estimate the elasticity of substitution.

The first set of instruments are local amenities that could affect labor supply developed by Albouy et al. (2016). They include measures of the slope, elevation, relative humidity, average dew point, average precipitation, and average sunlight for each local area. We also include multiple measures of temperature. The first measures are the number of heating degree days (HDD) and cooling degree days (CDD). HDD measures how cold a location is, and is defined as the sum of the difference between 65F and each day's mean temperature, for all days colder than 65F. CDD is a measure of how hot a location is, and is defined as the sum of the difference between each day's mean temperature and 65F, for all days warmer than 65 F. In addition, we include a set of temperature day bins which bin the average number of days in a year over 30 years that the average temperature (mean of minimum and maximum temperature) lie within the bin. We include 6 bins of 10 degrees Centigrade each.

The amenities in Albouy et al. (2016) were collected at the PUMA level. We aggregate them to the commuting zone level by taking an average across PUMAs in the same commuting zone, weighting PUMAs by their population in the commuting zone. We do not have amenities for Alaska and Hawaii, and all specifications exclude these states.

The second instrument, from Bartik (1991), is based upon the differential impact of national level shocks to industry employment across locations. Positive national shocks to an industry should increase labor demand and wages more in areas with high concentrations of that industry. Formally, the predicted growth rate in employment for a given location is the sum across industries of the product of the local employment share of this industry and the 5 or 10 year change in national level employment for that industry. We use the Longitudinal Business Database, which contains all US establishments, to construct these instruments.

The implicit assumption here is that changes in industry shares at the national level are independent of local manufacturing plant productivity. To help ensure that this assumption holds, we exclude manufacturing industries from the labor demand instrument. We calculate the instrument defining locations by commuting zones and industries at the SIC 4 digit level, or NAICS 6 digit level, depending upon the years. We drop industries with national employment of less than 100 people as likely data errors.

We also use a second set of labor demand instruments from Beaudry et al. (2012). The first instrument is the interaction of predicted changes in industry employment shares and industry initial wage premia. The second instrument is the interaction of national changes in industry wage premia and predicted future industry employment shares. We also exclude manufacturing industries from these instruments. National wage premia for an industry are calculated as the mean log payroll to employment ratio across the entire LBD for a given year.

The main complication with constructing the Bartik and BGS instruments is that industry definitions change over time; in 1987, when industry definitions switch from 1972 industry definitions to 1987 industry definitions, and in 1997, when industry definitions switch from the 1987 SIC definitions to NAICS definitions. Thus, we often cannot construct exact 10 year instruments because industry definitions are not consistent over time. Instead, we use 10 year instruments for 1987, 1997, and 2002, and 5 year instruments and their lag for 1992 and 2007. For 1987, the instrument used is from 1977 to 1986; for 1997, from 1987 to 1997; and for 2002, from 1992 to 2001. For 1992, we use the 1982 to 1986 and 1987 to 1992 instruments. For 2007, we use the 1997 to 2001 and 2002 to 2007 instruments.

B.5 Homogeneous Products

We use six homogeneous products: Boxes, Bread, Coffee, Concrete, Processed Ice and Plywood.[1] All of the products are defined as in Foster et al. (2008). We use data from 1987-1997 as capital data was imputed before 1987 for non-ASM plants, although we do not use data for 1992 for Processed Ice (because of data errors), 1987 for Boxes (because of a product definition change), and 1997 for Concrete (because quantity data was not recorded). We remove Census balancing codes imputed by the Census to make product level data add up to overall revenue data in cases where we can identify them. We also remove receipts for contract work, miscellaneous receipts, resales of products, and products with negative values.

We then remove all plants for which the product's share of plant revenue (measured after removing the balancing codes and other items mentioned above) is less than 50 percent. For each product, we have measures of both total quantity produced and revenue, which allows us to calculate product price as revenue over quantity. We delete all plants for which the ratio of product price to median product price is between .999 and 1.001, as these plants likely have quantity data

[1] Foster et al. (2008) examine 5 additional products: Carbon Black, Flooring, Gasoline, Block Ice, and Sugar; small samples in the years we study preclude this analysis.

imputed by the Census. We also remove plants with prices greater than ten times the median price or less than one-tenth the median price as potential mismeasured outliers.

C Micro Capital–Labor Elasticity Estimates

In this section, we examine differences in the plant capital-labor substitution elasticity across plants in multiple ways, including across age cohorts, quantiles of the plant capital cost to labor cost ratio, and industries.

C.1 Cohort Level Estimates

We first examine whether the plant capital-labor substitution elasticity varies across age cohorts. The elasticity could vary across age cohorts for multiple reasons. First, different vintages of plant technology could have different substitution elasticities. Second, exit of plants over time could lead to a changing average plant level elasticity.

We examine the plant level elasticity by 5 year age cohorts in Table C.1. Because we exclude plants entering in the Census year from our analysis, the first cohort is of plants between one to four years of age. The last cohort for each Census year also includes all plants operational before 1972, as the year of plant birth is censored at 1972.

Overall, we find fairly small differences in elasticities across age cohorts. Across all years and age cohorts, the smallest elasticity is 0.16 and largest 0.62. Moreover, across Census years, there is no clear pattern to differences in the elasticity across age cohorts. For 1987, 1997, and 2007, the cohort of the youngest plants does have an elasticity about 0.15 to 0.25 larger than the next youngest cohort. But, there is little difference between these cohorts for 1992 and 2002. In general, thus, we find no evidence for substantial differences in plant elasticities across age cohorts.

Table C.1 Age Cohort-Level Estimates of the Plant Capital-Labor Substitution Elasticity

	1-4	5-9	10-14	15-19	20-24	25-29	30-34
1987	0.62 *(0.05)*	0.37 *(0.05)*	0.40 *(0.04)*				
1992	0.45 *(0.04)*	0.42 *(0.03)*	0.41 *(0.04)*	0.52 *(0.05)*			
1997	0.40 *(0.06)*	0.21 *(0.06)*	0.21 *(0.06)*	0.16 *(0.07)*	0.36 *(0.08)*		
2002	0.25 *(0.09)*	0.26 *(0.07)*	0.31 *(0.08)*	0.37 *(0.08)*	0.33 *(0.10)*	0.34 *(0.10)*	
2007	0.57 *(0.07)*	0.43 *(0.06)*	0.45 *(0.07)*	0.45 *(0.07)*	0.38 *(0.08)*	0.41 *(0.08)*	0.45 *(0.06)*

Note: Standard errors are in parentheses. The table contains estimates of the elasticity of substitution by age cohort. Our measures of age are truncated, as we do not observe year of plant birth for plants operational before 1972. Thus, the oldest age cohort for each year also includes plants operational before 1972. All regressions include 4 digit SIC or 6 digit NAICS industry fixed effects, age fixed effects, and a multiunit status indicator and have standard errors clustered at the commuting zone level. Wages are estimated using data on workers from the Population Censuses and as defined in the text.

C.2 Non-CES Production Functions

We examine the case of local elasticities empirically by allowing elasticities to vary by quantile. Because we need to control for quantile-invariant industry fixed effects, we use the two-step approach of Canay (2011) to estimate quantile elasticities. We estimate the elasticity at the 10th through 90th quantile both among all plants and among plants within each 2-digit industry for each year. The elasticity varies across quantiles in an inverse U shape, with elasticities close to zero at the bottom quantiles, a peak close to the median, and then a slight fall for high quantiles. Web Appendix C.3 contains further details of the differences across quantiles and the estimation approach.

We then obtain $\bar{\sigma}_n$ by assigning each plant the elasticity for its closest conditional quantile. In Table C.2, column (2) reports estimates of the average $\bar{\sigma}_n$ under the assumption that elasticities at each quantile are the same across industries, while column (3) allows the these quantile elasticities to vary across industries. The average baseline OLS estimate across years is 0.39 (column (1)), compared to 0.45 using common quantile elasticities and 0.47 using separate quantile elasticities for each industry. Thus, the conditional quantile approach for allowing local elasticities increases our estimates of the aggregate elasticity slightly.

A second approach is to use plants' capital shares as the dependent variable instead of the logarithm of the ratio of capital cost to labor cost. Our goal is to estimate an approximation to $\bar{\sigma}_n \equiv \sum_{i \in I_n} \frac{\alpha_{ni}(1-\alpha_{ni})\theta_{ni}}{\sum_{i' \in I_n} \alpha_{ni'}(1-\alpha_{ni'})\theta_{ni'}} \sigma_{ni}$.

Consider the following regression equation:

$$\alpha_{nic} = \gamma_n + \beta_n \ln w_c + \epsilon_{nic}. \tag{C.1}$$

Here, β_n is an estimate of how the average capital share in a location covaries with relative factor prices in the location, i.e.,

$$\beta_n \approx \frac{\mathrm{d}\mathbb{E}[\alpha_{nic}]}{\mathrm{d}\ln w/r}. \tag{C.2}$$

In Web Appendix C.4, we show that, to a first order approximation, the estimator $\hat{\beta}_n$ converges to a weighted average of terms $\alpha_{nic}(1-\alpha_{nic})(\sigma_{nic}-1)$

$$\hat{\beta}_n \xrightarrow{p} \sum_c \sum_{i \in I_{nc}} \rho_{nic}\alpha_{nic}(1-\alpha_{nic})(\sigma_{nic}-1) \tag{C.3}$$

where I_{nc} are the set of plants in industry n in location c and the weights $\rho_{nic} = \frac{(\ln w_c - \overline{\ln w})^2}{\sum_{\tilde{c}} \sum_{\tilde{i} \in I_{n\tilde{c}}} (\ln w_{\tilde{c}} - \overline{\ln w})^2}$ sum to one.

Given our estimate for β, we compute $\bar{\sigma}_n$ using $\hat{\bar{\sigma}} - 1 = \frac{\hat{\beta}}{(1-\chi)\alpha(1-\alpha)}$. Column (4) of Table C.2 presents these estimates pooling across industries within the manufacturing sector.[2] The average elasticity across years is 0.44.[3]

Compared to our goal of estimating $\bar{\sigma}_n$, however, β_n weighs observations by ρ_{ni} instead of the cost weights θ_{ni}. First, plants in locations with more extreme wages are weighted more heavily, as is typical for least squares estimators. We have verified in Monte-Carlo simulations that this weighting does not lead to a significant bias; see Web Appendix C.4 for details. Second, we do not weight by θ_{ni}. To address this latter concern, we estimate (C.1) weighting each observation by θ_{ni}.

[2]In Web Appendix C.4, we also estimate $\bar{\sigma}_n$ separately for each industry and then take the appropriate average. In addition, we pursue an instrumental variables specification using the instruments used in Section 3.3. Estimates are quantitatively similar across specifications.

[3]The regression in column (4) of Table C.2 differs from (C.1) in that it includes the controls for a vector of plant characteristics X_{nic} (detailed in Table I): $\alpha_{nic} = \beta_n \ln w_c + \gamma X_{nic} + \epsilon_{nic}$. We show in Web Appendix C.4 that, in this case, the estimator converges in probability to $\hat{\beta}_n \xrightarrow{p} \sum_c \sum_{i \in I_{nc}} \rho^*_{nic}\alpha_{nic}(1-\alpha_{nic})(\sigma_{nic}-1)$ where the weights are $\rho^*_{nic} = \frac{\ln w^*_{nic}(\ln w_c) - \overline{\ln w})}{\sum_{\tilde{c}} \sum_{\tilde{i} \in I_{n\tilde{c}}} \ln w^*_{n\tilde{i}\tilde{c}}(\ln w_{\tilde{c}} - \overline{\ln w})}$ and $\ln w^*_{nic}$ are the residuals from a regression of $\ln w_c$ on X_{nic}.

However, we show analytically and confirm in our Monte-Carlo that this weighting also introduces an upward bias in elasticity estimates: larger plants tend to be more capital intensive which means that the weights are correlated with the error term. Thus, we view the weighted estimate as an upper bound on $\bar{\sigma}_n$. Column (5) shows that these estimates average 0.51 across years.

Given the narrow range of estimates in Table C.2, we do not believe that assuming a constant plant-level elasticity is a first order issue for our aggregation framework.

Table C.2 Non-CES Estimates of Average Plant Capital-Labor Substitution Elasticity

	(1) Baseline	(2) Quantile Sector Level	(3) Quantile Industry Level	(4) Avg Capital Share Unweighted	(5) Avg Capital Share Weighted
1987	0.43	0.46	0.47	0.45	0.56
1992	0.48	0.49	0.54	0.52	0.57
1997	0.34	0.39	0.45	0.43	0.58
2002	0.34	0.40	0.43	0.41	0.44
2007	0.38	0.52	0.45	0.40	0.38

Note: The table contains five specifications. All specifications average across separate plant elasticity of substitution for each industry using the cross industry weights used for aggregation. In (1), we estimate a separate OLS estimate using our baseline estimation strategy as in Section 3.3. In (2) and (3), we estimate separate elasticities for the 10th to the 90th quantiles using the two step estimation procedure of Canay (2011); (2) assumes a common estimate for all of manufacturing and (3) separate quantile elasticities for each 2 digit SIC or 3 digit NAICS industry. In (4) and (5), we estimate (C.1) by having the capital share as the dependent variable; (4) does not weight the data, while (5) weights plants by their total cost of capital and labor.

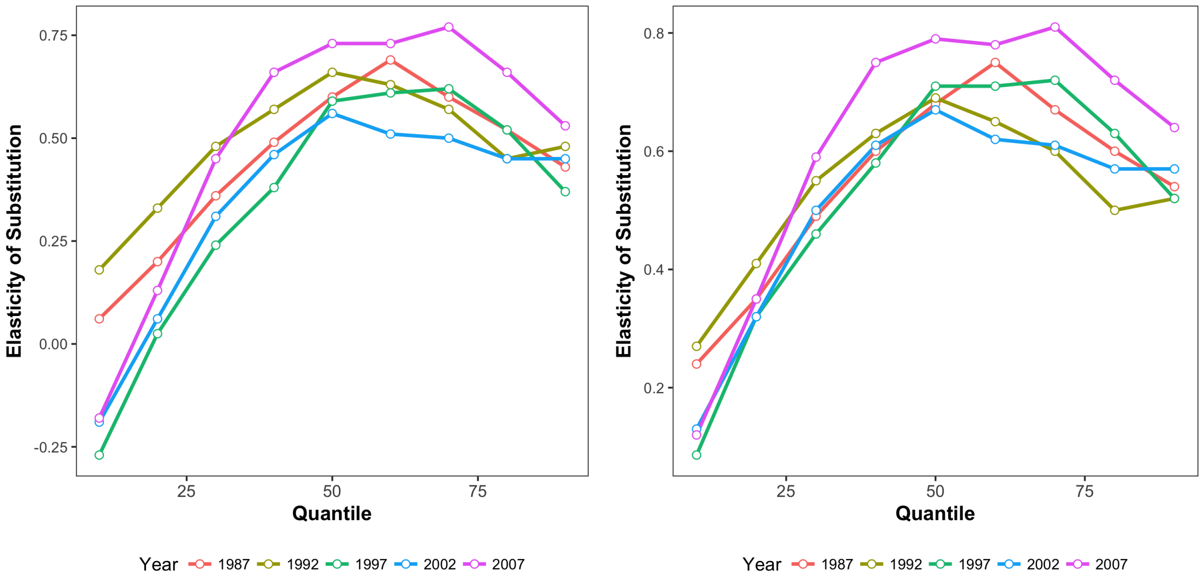
All regressions include industry fixed effects, age fixed effects, and a multi-unit status indicator. Wages used are the average log wage for the commuting zone, computed as wage and salary income over total number of hours worked adjusted for differences in worker characteristics using the Population Censuses.

C.3 Quantile Estimates

Our baseline econometric estimates assume that the effect of factor prices on relative factor costs is a pure location shift; that is, when moving from low wage commuting zones to high wage commuting zones, the entire distribution of the capital-labor ratio shifts uniformly. In this section, we examine this assumption through quantile estimates of the elasticity of substitution.

An immediate difficulty with any quantile estimate is that we have to control for hundreds of industry fixed effects. We do so through the two step estimation procedure of Canay (2011). We assume that the industry fixed effects are quantile invariant, and so are pure location shift effects. We first estimate our baseline OLS regressions in order to remove the estimated industry effects from the capital cost to labor cost ratio. We then estimate quantile regressions of the residual capital cost to labor cost ratio on the local area wage as well as the age and multiunit status controls.

We estimate these quantile regressions for the 10th through 90th quantile, both for all plants in manufacturing and by 2 digit SIC or 3 digit NAICS industry. Figure C.1 depicts these estimates for all plants in manufacturing across Census years; the left figure uses Population Census based wages while the right figure uses LBD based wages. We find a similar story across years and wage definitions. Moving from low wage commuting zones to high wage commuting zones, the



(a) Population Census Based Wages

(b) LBD Based Wages

Figure C.1 Quantile Estimates of the Plant Capital-Labor Elasticity of Substitution

Note: Both figures depict the elasticity of substitution estimated by quantile for each Census year. The left figure is based on wages from worker data from the Population Censuses, while the right figure is based on wages from establishment data from the LBD. Both are as defined in the text.

capital-labor ratio increases much more for the right tail of high capital-labor ratio plants than for the left tail of low capital-labor ratio plants. The elasticity tends to peak between the 50th and 70th quantile and then decrease slightly. For example, for the worker based estimates in 1987, our estimated elasticities are 0.06 at the 10th quantile, 0.2 at the 20th quantile, 0.6 at the median and 70th quantile, and 0.43 at the 90th quantile.

In order to construct an average plant level elasticity, we assign the associated quantile estimate to plants whose quantile of the capital cost to labor cost ratio distribution relative to the rest of its industry is close to the quantile estimate. Thus, we assign the 10th quantile estimate to plants between the 0th and 15th quantile on the plant capital cost to labor cost ratio, the 20th quantile estimate to plants between the 15 and 25 quantile on the plant capital cost to labor cost ratio, etc.

In Table C.3, we report estimates of the averaged quantile estimates across industries. The Sector Level estimates are based upon quantile elasticities estimated at the manufacturing level, and then averaged within and across industries to the manufacturing level; the Industry level estimates are based upon quantile elasticities at the industry level, and then averaged within and across industries to the manufacturing level. Estimates in Table C.3 use both Population Census based and LBD based wages. The quantile based estimates range from 0.39 to 0.54 using Population Census based wages, and from 0.54 to 0.63 using LBD based wages, and so are slightly higher than our baseline estimates.

C.4 Capital Share as a Dependent Variable

Consider the regression

$$\alpha_i = \beta_0 + \beta\omega_{cz(i)} + \epsilon_i. \tag{C.4}$$

Table C.3 Average Quantile Estimates of Average Plant Capital-Labor Substitution Elasticity

Quantile Level	Sector	Industry	Sector	Industry
1987	0.46	0.47	0.56	0.57
1992	0.49	0.54	0.54	0.59
1997	0.39	0.45	0.55	0.60
2002	0.40	0.43	0.54	0.56
2007	0.52	0.45	0.63	0.59
Wage	Pop Census		LBD	

Note: The table contains four specifications. All specifications average across separate plant elasticity of substitution for each industry using the cross industry weights used for aggregation. All specifications are based on estimates of separate elasticities for the 10th to the 90th quantiles estimated using the two step estimation procedure of Canay (2011); the first and third specification assumes a common estimate for all of manufacturing and the second and fourth specification separate quantile elasticities for each 2 digit SIC or 3 digit NAICS industry.
All regressions include industry fixed effects, age fixed effects, and a multi-unit status indicator. Wages used are the average log wage for the commuting zone, computed as wage and salary income over total number of hours worked adjusted for differences in worker characteristics from the Population Censuses in the first and second specifications and as payroll/number of employees at the establishment level from the LBD in the third and fourth specifications.

Estimating (C.4) using OLS yields the following estimator

$$\hat{\beta} = \frac{\sum_i \left(\omega_{cz(i)} - \bar{\omega}\right)\left(\alpha_i - \bar{\alpha}\right)}{\sum_i \left(\omega_{cz(i)} - \bar{\omega}\right)^2}$$

where the constants $\bar{\alpha} \equiv \frac{1}{|I|}\sum_i \alpha_i$ and $\bar{\omega} \equiv \frac{1}{|I|}\sum_i \omega_{cz(i)}$.

Consider the function $\alpha_i(\omega)$, which is what i's capital share would be with relative factor prices ω so that, abusing notation, $\alpha_i = \alpha_i\left(\omega_{cz(i)}\right)$. A first order approximation of $\alpha_i(\bar{\omega})$ around $\omega_{cz(i)}$ yields

$$\alpha_i = \alpha_i\left(\omega_{cz(i)}\right) \approx \alpha_i(\bar{\omega}) + \alpha_i\left(1-\alpha_i\right)(\sigma_i - 1)(\omega_{cz(i)} - \bar{\omega}) + O\left(\left(\omega_{cz(i)} - \bar{\omega}\right)^2\right) \tag{C.5}$$

Combining these equations and rearranging gives

$$\hat{\beta} = \frac{\sum_i \left(\omega_{cz(i)} - \bar{\omega}\right)^2 \alpha_i(1-\alpha_i)(\sigma_i - 1)}{\sum_i \left(\omega_{cz(i)} - \bar{\omega}\right)^2} + \frac{\sum_i \left(\omega_{cz(i)} - \bar{\omega}\right)\left(\alpha_i\left(\bar{\omega}\right) - \bar{\alpha}\right)}{\sum_i \left(\omega_{cz(i)} - \bar{\omega}\right)^2} \tag{C.6}$$

Our baseline regressions (without using an instrument) implicitly assumed that a plant's technology is independent of the local wage. We first proceed under the assumption that this assumption remains valid. We then discuss a corresponding approach using instrumental variables.

Since $E\left[\left(\omega_{cz(i)} - \bar{\omega}\right)\left(\alpha_i(\bar{\omega}) - \bar{\alpha}\right)\right] = 0$, the second term of (C.6) converges in probability to zero, so that

$$\hat{\beta} \xrightarrow{p} \sum_i \alpha_i(1-\alpha_i)\theta_i(\sigma_i - 1) + \sum_i (\rho_i - \theta_i)\alpha_i(1-\alpha_i)\theta_i(\sigma_i - 1)$$

where $\rho_i = \frac{(\omega_{cz(i)} - \bar{\omega})^2}{\sum_{\tilde{i}}(\omega_{cz(\tilde{i})} - \bar{\omega})^2}$. We do not have a strong reason to believe that the final term is positive or negative, as it is difficult to know how $(\rho_i - \theta_i)$ covaries with $\omega_{cz(i)}$. In any case, our Monte

Carlo exercise described below suggests that the bias is likely to be small.

Weighted Regression

Consider next estimating (C.4) using weighted least squares with weights θ_i. This yields the following estimator

$$\hat{\beta}_\theta = \frac{\sum_i \theta_i \left(\omega_{cz(i)} - \bar{\omega}\right)(\alpha_i - \bar{\alpha})}{\sum_i \theta_i \left(\omega_{cz(i)} - \bar{\omega}\right)^2}$$

where the constants $\bar{\alpha} \equiv \sum_i \theta_i \alpha_i$ and $\bar{\omega} \equiv \sum_i \theta_i \omega_{cz(i)}$. Using the first order approximation of $\alpha_i(\bar{\omega})$, this can be written as

$$\hat{\beta}_\theta = \sum_i \alpha_i(1-\alpha_i)\theta_i(\sigma_i - 1) + \sum_i (\rho_i - \theta_i)\alpha_i(1-\alpha_i)\theta_i(\sigma_i - 1) + \frac{\sum_i \left(\omega_{cz(i)} - \bar{\omega}\right)\theta_i\left(\alpha_i(\bar{\omega}) - \bar{\alpha}\right)}{\sum_i \left(\omega_{cz(i)} - \bar{\omega}\right)^2 \theta_i} \quad \text{(C.7)}$$

where $\rho_i = \frac{(\omega_{cz(i)} - \bar{\omega})^2 \theta_i}{\sum_{\tilde{i}} (\omega_{cz(\tilde{i})} - \bar{\omega})^2 \theta_{\tilde{i}}}$.

As discussed above, our Monte Carlo exercise suggests that the bias introduced by the second term is small.

We now argue that the third term is likely to be positive, leading to an upward bias, i.e., the estimate will tend to overstate $\bar{\sigma}$. To see why, note that it is well known larger plants tend to have higher capital shares, even within narrowly defined industries. Thus we expect $\sum \theta_i \left(\alpha_i\left(\bar{\omega}\right) - \bar{\alpha}\right) > 0$. An increase in ω raises θ_i more when $\alpha_i(\bar{\omega})$ is larger, i.e., when $\left(\alpha_i\left(\bar{\omega}\right) - \bar{\alpha}\right)$ is larger. Thus the positive covariance would be strengthened. Conversely, a reduction in ω weakens the covariance. Together, these imply that the second term is likely to be positive, so our estimator of $\bar{\sigma}$ is likely to be biased upwards.

Monte Carlo

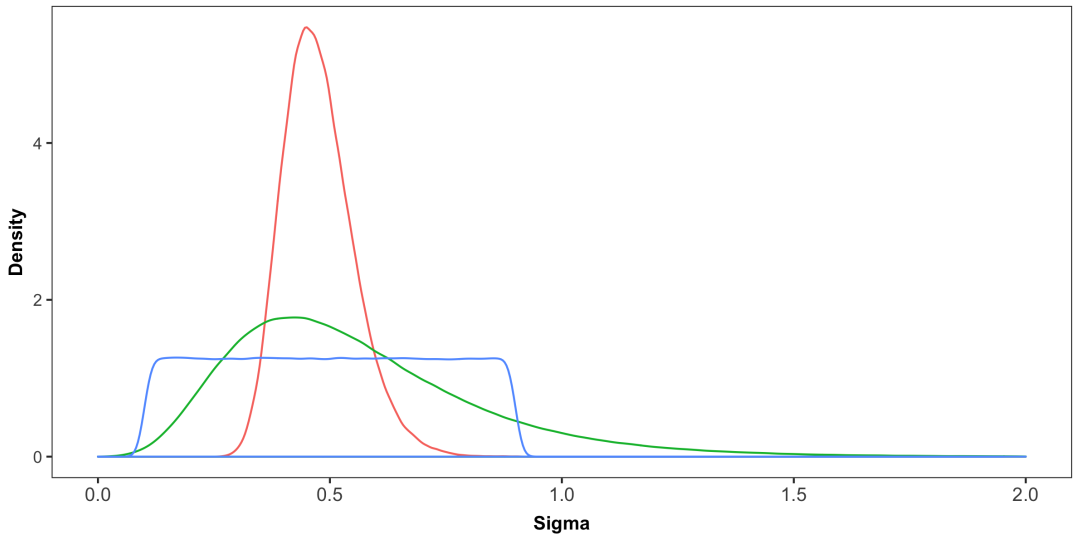
We then examine these biases using Monte Carlo simulations. We simulate an economy with 700 locations that each contain 100 plants. We normalize the rental rate to 1 and draw the natural log of each location's wage from a uniform (0,1) distribution. We assume that each plant produces using the CES production technology $Y_i = \left[(A_i K_i)^{\frac{\sigma_i - 1}{\sigma_i}} + (B_i L_i)^{\frac{\sigma_i - 1}{\sigma_i}}\right]^{\frac{\sigma_i}{\sigma_i - 1}}$ with an idiosyncratic elasticity of substitution drawn from a fixed distribution and an isoelastic demand curve with demand elasticity of 3. We also draw technology parameters A_i and B_i from a joint lognormal. We normalize the mean of A_i to 1, and choose the mean of B_i, the variances of A_i and B_i as well as their covariance to match the following four moments: an aggregate capital share of 0.3, a value of χ of 0.1, the 90-10 ratio of marginal cost across plants of 2.7, and the coefficient of a regression of $\log(\frac{\alpha_i}{1-\alpha_i})$ on $\log \theta_i$ (weighting by θ_i) of 0.08.[4]

We examine three parameterizations of the distribution of σ_i across plants; the densities of σ_i for each parameterization are depicted in Figure C.2 below. All three distributions are engineered to have a median value of σ_i close to 0.5. The first parameterization in red ("Low Variance

[4]Figure 5 depicts the aggregate share for the manufacturing sector over time, and Figure 1 values of χ across industries. Table 1 in Syverson (2004) examines dispersion in productivity (our value corresponds to the 90-10 ratio in TFP computed using plant specific input elasticities). Table 3 in Raval (2019) the coefficient of regressions of the capital share to labor share ratio on value added, weighting by value added, with estimates ranging from 0.05 to 0.09 using the Census of Manufactures across years, and 0.06 to 0.11 using the Annual Survey of Manufactures.

LogNormal") is drawn from a lognormal draw from Lognormal(-1,0.2^2) plus 0.1 (where -1 is the mean of the underlying normal distribution, and 0.2 the standard deviation of the underlying normal distribution); its density is concentrated around the median, with 98% of the distribution between 0.33 and 0.69. The second parameterization in green ("High Variance LogNormal") is drawn from a lognormal draw with Lognormal(-0.5,0.4^2) minus 0.1. It has a distribution that is somewhat skewed to the right, with 5% of the distribution above 1.07 and 1% above 1.44. Finally, the third parameterization ("Uniform") in blue is a uniform draw between 0.1 and 0.9.

Figure C.2 Density of σ



Note: This graph depicts the density of σ_i for three different parameterizations of σ. In red, the parameterization is 0.1 + a lognormal draw with Lognormal(-1,0.2^2). In green, the parameterization is -0.1 + a lognormal draw with Lognormal(-0.5,0.4^2). In blue, the parameterization is a uniform draw between 0.1 and 0.9.

We then run 200 simulations for each of the three parameterizations of the distribution of σ_i. Table C.4 contains these estimates. Column (2) reports the true weighted average $\bar{\sigma}$, the object we are trying attempting to recover. Column (3) reports an unweighted average of σ_i. Columns (4) and (5) report estimates of $\bar{\sigma}$ that correspond to columns (4) and (5) of Table C.2 and are derived from estimating α_i on the log of the wage in the plant's location. Column (4) reports the coefficient on an unweighted regression; column (5) weights by θ_i.

The unweighted average σ_i and estimates using the unweighted regression are, in general, fairly close to $\bar{\sigma}$. For the high variance lognormal parameterization of σ_i, the differences are the largest at 0.62 for $\bar{\sigma}$, compared to 0.56 for the unweighted average of σ_i and 0.54 for the unweighted regression. Across all three parameterizations, estimates using the weighted regression overstate the elasticity by about 0.2, as expected.

Covariates

Finally, consider the regression

$$\alpha_i = \beta\omega_{cz(i)} + \gamma x_i + \epsilon_i. \tag{C.8}$$

Table C.4 Monte Carlo Estimates of Average Plant Level Substitution Elasticity

	$\bar{\sigma}$	Average σ_i	Unweighted Regression	Weighted Regression
Low Variance LogNormal	0.48	0.48	0.51	0.71
High Variance LogNormal	0.62	0.56	0.54	0.82
Uniform	0.53	0.50	0.55	0.71

Note: The table contains four specifications based on 200 simulations with three different parameterizations of σ_i: the Low Variance LogNormal parameterization is 0.1 + a lognormal draw with Lognormal(-1,0.2^2), the High Variance LogNormal parameterization is -0.1 + a lognormal draw with Lognormal(-0.5,0.4^2), and the Uniform parameterization is a uniform draw between 0.1 and 0.9. For each, we report the average value of $\bar{\sigma}$, an unweighted average of σ_i, and estimates of the average σ from an unweighted and weighted regression (with cost weights) of α_i on the log wage.

where x_i is a set of characteristics of plants i such as age. Our identifying assumption is that for any fixed wage level $\bar{\omega}$, $\text{Cov}\left(\omega_{cz(i)}, \alpha_i(\bar{\omega})|x_i\right) = 0$.[5] Using the Frisch-Waugh-Lovell Theorem, we can express the estimator as

$$\hat{\beta} = \frac{\sum_i \omega_i^* \alpha_i}{\sum_i (\omega_i^*)^2} \tag{C.9}$$

where ω_i^* are the residuals after regressing $\omega_{cz(i)}$ on x_i. Note that $\sum_i \omega_i^* = 0$ by construction. Using same approximation as (C.5) and noting that $\sum_i (\omega_i^*)^2 = \sum_i \omega_i^*(\omega_{cz(i)} - \bar{\omega})$, we have

$$\begin{aligned}
\hat{\beta} &\approx \frac{\sum_i \omega_i^* \left[\alpha_i(1-\alpha_i)(\sigma_i - 1)(\omega_{cz(i)} - \bar{\omega}) + \alpha_i(\bar{\omega})\right]}{\sum_i \omega_i^* \left(\omega_{cz(i)} - \bar{\omega}\right)} \\
&= \frac{\sum_i \omega_i^*(\omega_{cz(i)} - \bar{\omega}) \left[\alpha_i(1-\alpha_i)(\sigma_i - 1)\right]}{\sum_i \omega_i^* \left(\omega_{cz(i)} - \bar{\omega}\right)} + \frac{\sum_i \omega_i^* \alpha_i(\bar{\omega})}{\sum_i \omega_i^* \left(\omega_{cz(i)} - \bar{\omega}\right)}
\end{aligned}$$

Our identifying assumption is that for any fixed wage level $\bar{\omega}$, $\text{Cov}\left(\omega_{cz(i)}, \alpha_i(\bar{\omega})|x_i\right) = 0$, or $\mathbb{E}[\omega_i^* \alpha_i(\bar{\omega})] = 0$. As a result, we have

$$\hat{\beta} \xrightarrow{p} \sum_i \rho_i^* \alpha_i(1-\alpha_i)(\sigma_i - 1)$$

where $\rho_i^* \equiv \frac{\omega_i^*(\omega_{cz(i)} - \bar{\omega})}{\sum_{i'} \omega_{i'}^* \left(\omega_{cz(i')} - \bar{\omega}\right)}$

Estimates using Capital Share as Dependent Variable

In Table C.5, we report estimates of the plant level elasticity of substitution using a specification in which the capital share is the dependent variable. The regression specification, for a given Census year, is:

$$\alpha_{nic} = \beta_n \log w_c + \gamma X_{nic} + \epsilon_{nic}. \tag{C.10}$$

The first set of estimates assumes that $\beta_n = \beta$ and so uses data from all of manufacturing. The second set of estimates allows β_n to vary by 2 digit SIC or 3 digit NAICS industries, and then averages these estimates across industries using cross industry weights.

[5] Or, for the IV specification, $\text{Cov}\left(z_{cz(i)}, \alpha_i(\bar{\omega})|x_i\right) = 0$.

The first and fourth columns are based on OLS estimates using worker based wages from the Population Censuses, and the second and fifth columns establishment wages from the LBD. The third and sixth columns are based on IV specifications using the amenity, Bartik, and BGS instruments. The first set of three columns do not weight the data, while the second set of three columns weight plants based on their total cost of capital and labor. The estimates using the capital share as a dependent variable range from 0.40 to 0.67 unweighted, and from 0.38 to 0.70 weighting with total cost weights. The unweighted estimates are only slightly higher than our baseline estimates, while the weighted estimates are slightly higher than the unweighted estimates.

Table C.5 Estimates of Plant Capital-Labor Substitution Elasticity using Capital Share As Dependent Variable

	Unweighted			Total Cost Weights		
Year	OLS	OLS	IV	OLS	OLS	IV
			Manufacturing Level Regressions			
1987	0.42 *(0.04)*	0.53 *(0.04)*	0.51 *(0.05)*	0.50 *(0.09)*	0.64 *(0.10)*	0.58 *(0.12)*
1992	0.55 *(0.02)*	0.60 *(0.02)*	0.46 *(0.09)*	0.53 *(0.10)*	0.63 *(0.09)*	0.54 *(0.05)*
1997	0.45 *(0.03)*	0.60 *(0.03)*	0.54 *(0.04)*	0.54 *(0.10)*	0.70 *(0.07)*	0.50 *(0.08)*
2002	0.48 *(0.04)*	0.60 *(0.03)*	0.56 *(0.04)*	0.43 *(0.08)*	0.60 *(0.06)*	0.42 *(0.07)*
2007	0.57 *(0.03)*	0.67 *(0.02)*	0.64 *(0.03)*	0.45 *(0.08)*	0.64 *(0.06)*	0.51 *(0.08)*
			Industry Specific Regressions			
1987	0.42	0.53	0.51	0.56	0.67	0.63
1992	0.52	0.56	0.53	0.57	0.64	0.54
1997	0.43	0.57	0.50	0.58	0.68	0.52
2002	0.41	0.53	0.47	0.44	0.57	0.44
2007	0.40	0.53	0.51	0.38	0.55	0.49
Wage	Pop Census	LBD	LBD	Pop Census	LBD	LBD

Note: All regressions include industry dummies, age fixed effects, and a multiunit status indicator. Instruments include amenity, Bartik, and BGS instruments. Wages used are the average log wage for the commuting zone. In the first and fourth columns, the wage is computed as wage and salary income over total number of hours worked adjusted for differences in worker characteristics from the Population Censuses; in all other cases, the wage is computed as payroll/number of employees at the establishment level from the LBD. The dependent variable is the capital share.

C.5 Dynamic Panel Estimates

In this section, we use the panel structure of our data in order to examine how individual plants respond to changes in factor prices. This adjustment may be slow; the long-run response to a factor price change should be larger than the short-run adjustment. We therefore use dynamic panel methods to examine both the unbalanced panel (which still requires plants that exist in at least three consecutive Census years), as well as the balanced panel of plants that exist in all five Census years.[6]

[6] Because our dynamic panel specification with two lags requires plants to be present for three consecutive Economic Censuses, there could be differences between the elasticity for these plants compared to the overall sample. These plants are likely to be different in some ways from the typical manufacturing plant; in particular, they may be older and larger, or belong to a multi-unit firm. We examine differences by age cohort in Web Appendix C.1 in the cross-section and do not find a clear gradient of the elasticity with age.

We estimate the following econometric model for plant i and time period t:

$$\log \frac{K_{itc}}{L_{itc}} = \rho_5 \log \frac{K_{it-5c}}{L_{it-5c}} + \rho_{10} \log \frac{K_{it-10c}}{L_{it-10c}} + \beta \log(w_{tc}/r_t) + \eta_i + \delta_t + \gamma_{n(i)} t + \epsilon_{itc} \qquad \text{(C.11)}$$

where η_i is an individual plant fixed effect, ρ_5 and ρ_{10} measure the degree of persistence in the capital-labor ratio through the five year and ten year lag of the capital-labor ratio, and β measures the short-run elasticity of substitution. We estimate this relationship in terms of the capital-labor ratio, and not the capital cost - labor cost ratio, so that the long run capital-labor elasticity is $\frac{\beta}{1-\rho_5-\rho_{10}}$.[7] Because we examine plants over time, we decompose the bias of plant i's technology into a plant fixed effect, η_i, a time fixed effect, δ_t, an 3-digit industry specific trend, $\gamma_{n(i)}t$, and a residual ϵ_{itc}.

We then use the Blundell-Bond panel data model to estimate this relationship.[8] We estimate two specifications; in the first, the wage-rental ratio is treated as exogenous after the time controls (i.e. exogenous with respect to ϵ_{itc}), while in the second specification, we use all of the instruments used earlier in Section 3.3 for the wage-rental ratio. We use local amenities as an instrument for the local wage level, while we use the Bartik and BGS shocks as instruments for both wage levels and changes. The wages we use are based on establishment data in order to match the same year as the Economic Census.

Table C.6 contains the estimates of these dynamic panel models. The first two columns report estimates for the unbalanced panel, the third and fourth columns for the balanced panel, and the fifth and sixth column for the unbalanced panel estimating the coefficients using second step GMM. The first three rows report the lag of the capital-labor ratio, the short-run elasticity, which is the coefficient on the wage-rental rate ratio, and the long-run elasticity, which is the short run elasticity divided by one minus the coefficient on the lag of the capital-labor ratio, across six specifications.

We start by estimating models with only the first lag of the capital-labor ratio, so $\rho_{10} = 0$. The coefficient on the lag of the capital-labor ratio is precisely estimated and ranges from 0.28 to 0.35, indicating substantially auto-correlation even over a 5 year time horizon. The estimates of the short run elasticity are fairly low, ranging from 0.06 to 0.22 across specifications, indicating a long run elasticities between 0.08 and 0.31. These long run elasticities are substantially lower than our cross-sectional estimates.

One of the main testable assumptions of the Blundell-Bond model with one lag is that there is no correlation between ϵ_{it} and ϵ_{it-10}. We can test this by examining the correlation between differenced residuals; while the autocorrelation is low (at about 0.045), we strongly reject the hypothesis that there is no correlation between errors two periods apart. Thus, we also estimate specifications including a second lag of the capital-labor ratio in the fourth through seventh rows

In Web Appendix C.4, we find that weighting by size leads to only slightly larger estimates of the elasticity, and, as discussed in Section 3.3, estimates using multi-unit plants tend to be similar to the full sample. Thus, we suspect that any sample selection bias is small.

[7] We measure the labor input at a plant as the wage bill divided by the local wage.

[8] Blundell-Bond uses system GMM with two equations. One moment condition is based on differencing (C.11) and then instrumenting with lagged terms, so $E\left[Z_{it-5}(\epsilon_{it} - \epsilon_{it-5})\right] = 0$. The second moment condition uses (C.11) directly but differences the instruments, so $E\left[(Z_{it} - Z_{it-5})\epsilon_{it}\right] = 0$. For example, in the differenced equation, we would instrument for the change in the capital-labor ratio with lagged values of the capital-labor ratio, while in the levels equation we would instrument for the lag of the capital-labor ratio with lagged values of changes in the capital-labor ratio. We also examined estimates using the Arellano-Bond model, which only uses the differenced equation. Unfortunately, with the Arellano-Bond model we have very little power to estimate the capital-labor elasticity in specifications where we instrument for wages, although we obtain similar estimates of the lagged capital-labor ratio.

of Table C.6. The coefficient on the first lag ranges from 0.31 to 0.34. The coefficient on the second lag is roughly one-fifth to one-fourth the magnitude of the second lag, ranging from 0.06 to 0.08, but remains strongly significant. Thus, dynamic adjustment does occur beyond a five year time horizon. However, the sharp reduction in the magnitude of the second lag gives us confidence that we do not need to include additional lags of the capital-labor ratio.

The short run elasticities in the specifications with two lags of the capital-labor ratio are considerably higher than those with one lag, with estimates between 0.15 and 0.36. These short run elasticities imply long run elasticities between 0.26 and 0.61.

Apart from the estimate of 0.61, however, these long run elasticities remain slightly below most of the estimates of the cross-sectional elasticity.

Table C.6 Dynamic Panel Estimates of the Plant Capital-Labor Substitution Elasticity

	(1) No Inst	(2) All Inst	(3) No Inst	(4) All Inst	(5) No Inst	(6) All Inst
Lag	0.28 *(0.003)*	0.31 *(0.004)*	0.34 *(0.004)*	0.35 *(0.004)*	0.28 *(0.007)*	0.31 *(0.006)*
SR Elasticity	0.13 *(0.05)*	0.07 *(0.07)*	0.12 *(0.05)*	0.06 *(0.08)*	0.22 *(0.12)*	0.08 *(0.08)*
LR Elasticity	0.18	0.10	0.18	0.08	0.31	0.11
Lag	0.31 *(0.004)*	0.32 *(0.005)*	0.34 *(0.004)*	0.34 *(0.005)*	0.31 *(0.008)*	0.33 *(0.007)*
Second Lag	0.06 *(0.005)*	0.07 *(0.005)*	0.08 *(0.005)*	0.07 *(0.006)*	0.07 *(0.008)*	0.07 *(0.007)*
SR Elasticity	0.18 *(0.04)*	0.21 *(0.09)*	0.15 *(0.04)*	0.36 *(0.14)*	0.27 *(0.09)*	0.21 *(0.08)*
LR Elasticity	0.29	0.34	0.26	0.61	0.43	0.35
Balanced	No	No	Yes	Yes	No	No
Two Step	No	No	No	No	Yes	Yes

Note: The table contains six specifications. In (1) and (2), we examine an unbalanced panel of plants in the Census of Manufactures for at least three consecutive Censuses between 1987 and 2007, while (3) and (4) examine the balanced panel. The first four specifications use one step GMM, while (5) and (6) use two step GMM on the unbalanced panel.

All specifications estimate the Blundell-Bond model, either assuming that the wage-rental rate ratio is exogenous, or instrumenting for it using amenity, Bartik, and BGS instruments. Instruments are as defined in the text. All specifications also include year effects and time trends for the 3 Digit NAICS industry reported in 1997 level as controls for biased technical change. The wage is the average log wage for the commuting zone, computed as payroll/number of employees at the establishment level using the LBD. The rental rate is the average rental rate between structures and equipment, weighting each by their respective capital stock. Standard errors, in parentheses, are clustered at the commuting zone level.

C.6 Estimates Including Spillover Wage

We examine the magnitude of spillovers from wages in nearby commuting zones by including both the commuting zone's own wage, and the average wage in other commuting zones in the same state. We construct this average spillover wage by weighting the local commuting zone wage in other commuting zones by their total manufacturing employment in our data. In Table C.7, we display both the coefficient on the local wage and the spillover wage (so, under our baseline model, the elasticity is one minus the coefficient on the local wage). The spillover wage effects are typically small, with most below 10% of the local wage effect, and not statistically significantly different from zero.

Table C.7 Estimates of Spillover Effects

	Local Wage	Spillover Wage	Local Wage	Spillover Wage
1987	-0.57 *(0.03)*	0.01 *(0.06)*	-0.5 *(0.03)*	-0.10 *(0.07)*
1992	-0.45 *(0.03)*	-0.05 *(0.04)*	-0.62 *(0.06)*	-0.19 *(0.13)*
1997	-0.62 *(0.03)*	0.18 *(0.05)*	-0.47 *(0.04)*	-0.18 *(0.09)*
2002	-0.48 *(0.03)*	0.03 *(0.05)*	-0.51 *(0.04)*	-0.07 *(0.09)*
2007	-0.69 *(0.05)*	-0.06 *(0.11)*	-0.39 *(0.02)*	-0.09 *(0.06)*
Wage	Pop Census		LBD	

Note: Standard errors are in parentheses. The table contains estimates of the coefficient on the local commuting zone wage as well as the coefficient on the spillover wage, measured as the average local wage in other commuting zones in the same state, averaged after weighting by total manufacturing employment. All regressions include 4 digit SIC or 6 digit NAICS industry fixed effects, age fixed effects, and a multiunit status indicator and have standard errors clustered at the commuting zone level. Wages are based upon Population Census or LBD data, depending upon the specification, and as defined in the text.

C.7 Estimates Using the Plant-Level Wage

In our baseline estimates, we use the local wage as the wage rate that the plant faces. In this section, we examine the alternative of using the plant-level wage. We first show the assumptions required for the regional wage or the plant wage to identify the elasticity. We then examine estimates using the plant level wage in light of our theoretical results.

We write all variables in log form. We first assume that the wage the plant faces for a unit of human capital, w_i^P, both reflects the local wage, w_i^L, and a plant specific compensating differential, S_i. The observed plant level wage, $\hat{w}_i^P$, also includes the amount of human capital per worker, τ_i. Thus, we have that:

$$\begin{aligned} w_i^P &= w_i^L + S_i \\ \hat{w}_i^P &= w_i^P + \tau_i \end{aligned}$$

Since we construct the local wage averaging across workers or establishments in a location, the plant specific compensating differential should be mean zero conditional on the local wage:

$$E\left[S_i|w_i^L\right] = 0$$

An implication of this is that the covariance of the local wage w_i^L and the plant compensating differential S_i should be zero: $Cov\left(w_i^L, S_i\right) = 0$.

The true model for the factor cost ratio is:

$$y_i = \beta w_i^P + \varepsilon_i$$

where y_i is the factor cost ratio and $\beta = \sigma - 1$. For simplicity in demonstrating the differences between the plant level wage and local wage, we assume that the error term ε_i is i.i.d and independent of right hand side variables and instruments.

Our baseline OLS uses the local wage. Under this setup, we have that:

$$\frac{Cov\left(y_i, w_i^L\right)}{Var\left(w_i^L\right)} = \beta\frac{Cov\left(w_i^P, w_i^L\right)}{Var\left(w_i^L\right)} = \beta\frac{Cov\left(w_i^L + S_i, w_i^L\right)}{Var\left(w_i^L\right)} = \beta$$

Thus, OLS using the local wage identifies the elasticity of substitution.

If we estimate OLS using the plant level wage, we have that:

$$\begin{aligned}\frac{Cov\left(y_i, \hat{w}_i^P\right)}{Var\left(\hat{w}_i^P\right)} &= \beta\frac{Cov\left(w_i^P, \hat{w}_i^P\right)}{Var\left(\hat{w}_i^P\right)} = \beta\frac{Cov\left(\hat{w}_i^P - \tau, \hat{w}_i^P\right)}{Var\left(\hat{w}_i^P\right)} \\ &= \beta\left(1 - \frac{Cov\left(\tau_i, \hat{w}_i^P\right)}{Var\left(\hat{w}_i^P\right)}\right)\end{aligned}$$

So long as measured wages are correlated with skill, so $Cov\left(\tau_i, \hat{w}_i^P\right)$, which is likely, then OLS with the plant level wage will bias estimates of β towards zero, and so biases estimates of the elasticity σ towards one.

Next, we could instrument for the plant level wage using the local wage. (Similar issues would apply for other instruments.) In that case, we have that:

$$\begin{aligned}\frac{Cov\left(y_i, w_i^L\right)}{Cov\left(\hat{w}_i^P, w_i^L\right)} &= \beta\frac{Cov\left(w_i^P, w_i^L\right)}{Cov\left(\hat{w}_i^P, w_i^L\right)} = \beta\frac{Var\left(w_i^L\right) + Cov\left(S_i, w_i^L\right)}{Var\left(w_i^L\right) + Cov\left(S, w_i^L\right) + Cov\left(\tau_i, w_i^L\right)} \\ &= \beta\frac{Var\left(w_i^L\right)}{Var\left(w_i^L\right) + Cov\left(\tau_i, w_i^L\right)}\end{aligned}$$

In this case, for identification of β, and so the elasticity σ, we require an additional assumption that $Cov\left(\tau_i, w_i^L\right) = 0$. That is, we need that the degree of skill at the plant is uncorrelated with the local wage. If areas with higher wages also have workers with higher human capital, this correlation would be positive; otherwise, it would be negative.

We then examine several specifications using the plant wage in Table C.8. Estimates using OLS regressions with the plant wage are much higher than our baseline estimates, ranging from 0.77 to 0.94, consistent with the expected bias from the correlation of plant level skill with the measured plant level wage. Using either local wages as instruments or our previous instruments, we get slightly lower estimates from 1987 through 1997. These estimates range between 0.25 and 0.35 using local wages, and between 0.14 and 0.4 using all of our three sets of instruments. Slightly lower estimates using these instruments with the plant level wage would be consistent with plants in higher wage areas using less skilled workers on average.

However, we obtain much lower estimates for 2002 and 2007 using instruments, with estimates between 0.01 and 0.05 using local wages and 0.13 and 0.14 using all three sets of instruments together. Elasticity estimates are negative for some of the specifications in these years. One explanation for the difference is that plant level wages for 2002 and 2007, unlike 1987 through 1997, included non-monetary compensation. Thus, compared to using the local wage, using the plant level wage results in lower and more variable estimates.

In Section 3.3.2, we found that estimated elasticities of substitution were higher after including firm fixed effects. One potential explanation for this result is that the plant level wage is less responsive to the local market wage within multiplant firms. We examine this explanation in Table C.9 by regressing the plant wage on the local wage for multiunit plants, and either including or excluding firm fixed effects. We indeed find that the correlation between the local wage and plant wage is lower after including firm fixed effects. For example, in the IV specifications, the plant wage increases, on average, by 54% with a 100% increase in the local wage without including firm fixed effects, compared to 48% after including firm fixed effects.

Table C.8 Estimates of the Capital-Labor Elasticity of Substitution Using Plant-Level Wages and Instruments

	OLS	Local Wage (Pop Census)	Local Wage (LBD)	Amenities	Bartik	BGS	All
1987	0.84 *(0.01)*	0.27 *(0.07)*	0.25 *(0.07)*	0.32 *(0.10)*	0.24 *(0.10)*	0.28 *(0.10)*	0.33 *(0.06)*
1992	0.84 *(0.01)*	0.35 *(0.06)*	0.3 *(0.05)*	0.5 *(0.07)*	0.22 *(0.08)*	0.28 *(0.08)*	0.38 *(0.05)*
1997	0.77 *(0.01)*	0.28 *(0.06)*	0.25 *(0.06)*	0.44 *(0.07)*	0.19 *(0.18)*	0.14 *(0.13)*	0.32 *(0.06)*
2002	0.94 *(0.02)*	0.04 *(0.11)*	0.01 *(0.10)*	0.33 *(0.14)*	-0.47 *(0.30)*	-0.11 *(0.14)*	0.14 *(0.09)*
2007	0.81 *(0.02)*	0.05 *(0.10)*	0.05 *(0.09)*	0.17 *(0.13)*	-0.13 *(0.15)*	0.05 *(0.11)*	0.13 *(0.07)*

Note: Standard errors are in parentheses. The table contains estimates of the elasticity of substitution using the plant-level wage, defined as total wage bill divided by total employment. The second column reports OLS results, wheas all other columns use an instrument: either the local wage using Population Census based wages, the local wage using LBD based wages, the amenity instruments, Bartik instruments, BGS instruments, or the amenity, Bartik, and BGS instruments combined. All regressions include 4 digit SIC or 6 digit NAICS industry fixed effects, age fixed effects, and a multiunit status indicator and have standard errors clustered at the commuting zone level. Instruments are as defined in the text.

Table C.9 Correlation of the Plant Wage with the Local Wage for Multi-Unit Plants

	Including Firm Fixed Effects			Excluding Firm Fixed Effects		
Year	OLS	OLS	IV	OLS	OLS	IV
1987	0.58 *(0.03)*	0.43 *(0.02)*	0.48 *(0.03)*	0.66 *(0.04)*	0.51 *(0.02)*	0.54 *(0.03)*
1992	0.71 *(0.03)*	0.53 *(0.02)*	0.57 *(0.03)*	0.86 *(0.03)*	0.66 *(0.02)*	0.70 *(0.03)*
1997	0.75 *(0.03)*	0.48 *(0.02)*	0.55 *(0.03)*	0.91 *(0.03)*	0.59 *(0.03)*	0.65 *(0.03)*
2002	0.66 *(0.02)*	0.42 *(0.02)*	0.45 *(0.03)*	0.75 *(0.03)*	0.49 *(0.02)*	0.51 *(0.03)*
2007	0.63 *(0.03)*	0.44 *(0.02)*	0.46 *(0.03)*	0.75 *(0.03)*	0.53 *(0.02)*	0.56 *(0.03)*
Wage	Pop Census	LBD	LBD	Pop Census	LBD	LBD

Note: All regressions include industry dummies and age fixed effects, and only use multiunit plants; the first three columns include firm fixed effects, while the second three columns exclude firm fixed effects. Instruments include amenity, Bartik, and BGS instruments. Local wages used are the average log wage for the commuting zone. In the first and fourth columns, the local wage is computed as wage and salary income over total number of hours worked adjusted for differences in worker characteristics using the Population Censuses; in all other cases, the wage is computed as payroll/number of employees at the establishment level using the LBD. The dependent variable is the plant level wage.

C.8 Plant Level Estimates by Industry

This section includes tables of plant capital-labor substitution elasticity estimates from Table C.10 to Table C.15.

Table C.10 Elasticities of Substitution between Labor and Capital for Two Digit SIC Industries Using Population Census Based Wages

Industry	1987	1992	1997	N (1987)
20: Food Products	0.60 *(0.06)*	0.69 *(0.08)*	0.63 *(0.09)*	≈ 11,200
22: Textiles	0.58 *(0.16)*	0.64 *(0.18)*	0.39 *(0.19)*	≈ 3,580
23: Apparel	0.94 *(0.10)*	0.73 *(0.05)*	0.36 *(0.09)*	≈ 12,800
24: Lumber and Wood	0.19 *(0.08)*	0.47 *(0.08)*	0.09 *(0.10)*	≈ 15,500
25: Furniture	0.19 *(0.10)*	0.43 *(0.10)*	-0.17 *(0.20)*	≈ 5,720
26: Paper	0.22 *(0.10)*	0.35 *(0.09)*	0.46 *(0.10)*	≈ 4,280
27: Printing and Publishing	0.45 *(0.04)*	0.34 *(0.05)*	0.26 *(0.07)*	≈ 27,800
28: Chemicals	0.31 *(0.12)*	0.26 *(0.12)*	0.09 *(0.13)*	≈ 7,040
29: Petroleum Refining	0.45 *(0.18)*	1.15 *(0.18)*	0.47 *(0.23)*	≈ 1,670
30: Rubber	0.45 *(0.12)*	0.44 *(0.11)*	0.23 *(0.10)*	≈ 8,630
31: Leather	0.70 *(0.22)*	0.54 *(0.19)*	0.45 *(0.29)*	≈ 1,000
32: Stone, Clay, Glass, Concrete	0.20 *(0.11)*	0.58 *(0.11)*	0.26 *(0.13)*	≈ 9,360
33: Primary Metal	0.40 *(0.12)*	0.32 *(0.10)*	0.21 *(0.17)*	≈ 4,380
34: Fabricated Metal	0.24 *(0.07)*	0.38 *(0.07)*	0.14 *(0.09)*	≈ 21,000
35: Machinery	0.47 *(0.05)*	0.47 *(0.06)*	0.38 *(0.08)*	≈ 26,100
36: Electrical Machinery	0.42 *(0.10)*	0.47 *(0.11)*	0.52 *(0.11)*	≈ 8,300
37: Transportation Equip	0.59 *(0.13)*	0.70 *(0.14)*	0.47 *(0.12)*	≈ 5,130
38: Instruments	0.61 *(0.11)*	0.39 *(0.10)*	0.41 *(0.11)*	≈ 4,680
39: Misc	0.33 *(0.10)*	0.25 *(0.11)*	-0.00 *(0.11)*	≈ 6,900

Note: All regressions include 4 digit SIC industry fixed effects, age fixed effects, and a multiunit status indicator and have standard errors clustered at the commuting zone level. Wages are based upon Population Census data and as defined in the text.

Table C.11 Elasticities of Substitution between Labor and Capital for Two Digit SIC Industries Using LBD Based Wages

Industry	1987	1992	1997	N (1987)
20: Food Products	0.68 *(0.05)*	0.74 *(0.07)*	0.74 *(0.07)*	≈ 11,200
22: Textiles	0.68 *(0.12)*	0.67 *(0.15)*	0.52 *(0.13)*	≈ 3,580
23: Apparel	0.99 *(0.07)*	0.78 *(0.05)*	0.52 *(0.07)*	≈ 12,800
24: Lumber and Wood	0.37 *(0.06)*	0.47 *(0.06)*	0.34 *(0.07)*	≈ 15,500
25: Furniture	0.30 *(0.08)*	0.42 *(0.09)*	0.17 *(0.13)*	≈ 5,720
26: Paper	0.36 *(0.08)*	0.43 *(0.09)*	0.17 *(0.13)*	≈ 4,280
27: Printing and Publishing	0.51 *(0.03)*	0.39 *(0.04)*	0.44 *(0.05)*	≈ 27,800
28: Chemicals	0.44 *(0.12)*	0.29 *(0.12)*	0.25 *(0.11)*	≈ 7,040
29: Petroleum Refining	0.50 *(0.15)*	1.04 *(0.17)*	0.57 *(0.18)*	≈ 1,670
30: Rubber	0.53 *(0.10)*	0.50 *(0.08)*	0.45 *(0.08)*	≈ 8,630
31: Leather	0.80 *(0.18)*	0.59 *(0.16)*	0.53 *(0.19)*	≈ 1,000
32: Stone, Clay, Glass, Concrete	0.39 *(0.09)*	0.68 *(0.09)*	0.44 *(0.09)*	≈ 9,360
33: Primary Metal	0.58 *(0.09)*	0.39 *(0.08)*	0.44 *(0.11)*	≈ 4,380
34: Fabricated Metal	0.40 *(0.06)*	0.43 *(0.06)*	0.38 *(0.06)*	≈ 21,000
35: Machinery	0.59 *(0.04)*	0.57 *(0.06)*	0.58 *(0.06)*	≈ 26,100
36: Electrical Machinery	0.54 *(0.09)*	0.59 *(0.09)*	0.61 *(0.07)*	≈ 8,300
37: Transportation Equip	0.64 *(0.10)*	0.69 *(0.12)*	0.65 *(0.09)*	≈ 5,130
38: Instruments	0.60 *(0.09)*	0.39 *(0.10)*	0.54 *(0.09)*	≈ 4,680
39: Misc	0.40 *(0.08)*	0.29 *(0.09)*	0.24 *(0.09)*	≈ 6,900

Note: All regressions include 4 digit SIC industry fixed effects, age fixed effects, and a multiunit status indicator and have standard errors clustered at the commuting zone level. Wages are based upon LBD data and as defined in the text.

Table C.12 Elasticities of Substitution between Labor and Capital for Two Digit SIC Industries Using All Instruments

Industry	1987	1992	1997	N (1987)
20: Food Products	0.65 *(0.06)*	0.62 *(0.09)*	0.67 *(0.09)*	≈ 11,200
22: Textiles	0.64 *(0.12)*	0.78 *(0.16)*	0.47 *(0.16)*	≈ 3,580
23: Apparel	0.93 *(0.09)*	0.77 *(0.05)*	0.39 *(0.07)*	≈ 12,800
24: Lumber and Wood	0.34 *(0.10)*	0.53 *(0.09)*	0.28 *(0.10)*	≈ 15,500
25: Furniture	0.23 *(0.08)*	0.35 *(0.11)*	0.02 *(0.20)*	≈ 5,720
26: Paper	0.28 *(0.10)*	0.41 *(0.08)*	0.40 *(0.10)*	≈ 4,280
27: Printing and Publishing	0.52 *(0.04)*	0.34 *(0.04)*	0.37 *(0.06)*	≈ 27,800
28: Chemicals	0.31 *(0.12)*	0.18 *(0.13)*	0.15 *(0.13)*	≈ 7,040
29: Petroleum Refining	0.49 *(0.19)*	1.13 *(0.22)*	0.51 *(0.23)*	≈ 1,670
30: Rubber	0.50 *(0.14)*	0.45 *(0.11)*	0.39 *(0.10)*	≈ 8,630
31: Leather	0.68 *(0.20)*	0.55 *(0.17)*	0.66 *(0.21)*	≈ 1,000
32: Stone, Clay, Glass, Concrete	0.23 *(0.12)*	0.64 *(0.13)*	0.29 *(0.13)*	≈ 9,360
33: Primary Metal	0.49 *(0.11)*	0.48 *(0.10)*	0.29 *(0.16)*	≈ 4,380
34: Fabricated Metal	0.38 *(0.07)*	0.41 *(0.07)*	0.31 *(0.09)*	≈ 21,000
35: Machinery	0.57 *(0.05)*	0.54 *(0.06)*	0.52 *(0.07)*	≈ 26,100
36: Electrical Machinery	0.48 *(0.11)*	0.59 *(0.10)*	0.63 *(0.09)*	≈ 8,300
37: Transportation Equip	0.76 *(0.14)*	0.60 *(0.13)*	0.62 *(0.12)*	≈ 5,130
38: Instruments	0.63 *(0.11)*	0.51 *(0.13)*	0.47 *(0.10)*	≈ 4,680
39: Misc	0.33 *(0.09)*	0.21 *(0.10)*	0.17 *(0.11)*	≈ 6,900

Note: All regressions include 4 digit SIC industry fixed effects, age fixed effects, and a multiunit status indicator and have standard errors clustered at the commuting zone level. Wages are based upon LBD data and as defined in the text. Instruments include amenity, Bartik, and BGS instruments together and are as defined in the text.

Table C.13 Elasticities of Substitution between Labor and Capital for Three Digit NAICS Industries Using Population Census Based Wages

Industry	1997	2002	2007	N (1997)
311: Food Products	0.51 *(0.07)*	0.69 *(0.09)*	0.70 *(0.08)*	≈ 15,000
312: Beverage	1.10 *(0.36)*	1.07 *(0.22)*	1.09 *(0.27)*	≈ 1,380
313: Textiles	0.35 *(0.25)*	0.04 *(0.19)*	-0.14 *(0.25)*	≈ 2,650
314: Textile Products	0.23 *(0.13)*	-0.02 *(0.20)*	0.70 *(0.14)*	≈ 4,130
315: Apparel	0.34 *(0.10)*	0.23 *(0.17)*	0.42 *(0.16)*	≈ 10,200
316: Leather	0.44 *(0.28)*	0.63 *(0.31)*	0.78 *(0.44)*	≈ 914
321: Wood Products	0.09 *(0.10)*	-0.38 *(0.15)*	-0.04 *(0.08)*	≈ 11,000
322: Paper	0.38 *(0.11)*	0.37 *(0.12)*	0.52 *(0.15)*	≈ 4,420
323: Printing	0.26 *(0.07)*	0.50 *(0.08)*	0.61 *(0.06)*	≈ 23,900
324: Petroleum Refining	0.47 *(0.23)*	0.33 *(0.24)*	0.42 *(0.21)*	≈ 1,620
325: Chemicals	0.12 *(0.13)*	0.08 *(0.14)*	0.06 *(0.14)*	≈ 8,370
326: Rubber	0.26 *(0.10)*	0.39 *(0.11)*	0.19 *(0.10)*	≈ 11,300
327: Stone, Clay, Glass, Concrete	0.25 *(0.13)*	0.00 *(0.13)*	0.39 *(0.11)*	≈ 10,600
331: Primary Metal	0.31 *(0.16)*	0.19 *(0.17)*	0.44 *(0.22)*	≈ 3,560
332: Fabricated Metal	0.33 *(0.07)*	0.45 *(0.10)*	0.45 *(0.06)*	≈ 37,700
333: Machinery	0.16 *(0.11)*	0.15 *(0.09)*	0.43 *(0.08)*	≈ 18,500
334: Computers	0.50 *(0.10)*	0.50 *(0.14)*	0.44 *(0.12)*	≈ 9,230
335: Electrical Equip	0.28 *(0.18)*	0.25 *(0.19)*	0.57 *(0.18)*	≈ 4,080
336: Transportation Equip	0.45 *(0.11)*	0.44 *(0.17)*	0.21 *(0.15)*	≈ 7,030
337: Furniture	-0.02 *(0.15)*	0.11 *(0.15)*	0.49 *(0.11)*	≈ 10,300
339: Misc	0.15 *(0.09)*	0.21 *(0.10)*	0.62 *(0.06)*	≈ 12,500

Note: All regressions include 6 digit NAICS industry fixed effects, age fixed effects, and a multiunit status indicator and have standard errors clustered at the commuting zone level. Wages are based upon Population Census data and as defined in the text.

Table C.14 Elasticities of Substitution between Labor and Capital for Three Digit NAICS Industries Using LBD Based Wages

Industry	1997	2002	2007	N (1997)
311: Food Products	0.64 *(0.05)*	0.75 *(0.06)*	0.76 *(0.06)*	≈ 15,000
312: Beverage	1.14 *(0.25)*	1.07 *(0.17)*	1.11 *(0.22)*	≈ 1,380
313: Textiles	0.52 *(0.18)*	0.23 *(0.14)*	0.12 *(0.20)*	≈ 2,650
314: Textile Products	0.38 *(0.10)*	0.24 *(0.15)*	0.76 *(0.12)*	≈ 4,130
315: Apparel	0.55 *(0.09)*	0.44 *(0.13)*	0.59 *(0.13)*	≈ 10,200
316: Leather	0.52 *(0.19)*	0.70 *(0.24)*	0.90 *(0.32)*	≈ 914
321: Wood Products	0.38 *(0.08)*	0.06 *(0.10)*	0.28 *(0.07)*	≈ 11,000
322: Paper	0.58 *(0.08)*	0.55 *(0.09)*	0.64 *(0.11)*	≈ 4,420
323: Printing	0.43 *(0.05)*	0.60 *(0.06)*	0.67 *(0.04)*	≈ 23,900
324: Petroleum Refining	0.57 *(0.18)*	0.37 *(0.18)*	0.69 *(0.16)*	≈ 1,620
325: Chemicals	0.29 *(0.10)*	0.23 *(0.10)*	0.24 *(0.12)*	≈ 8,370
326: Rubber	0.47 *(0.08)*	0.54 *(0.08)*	0.40 *(0.07)*	≈ 11,300
327: Stone, Clay, Glass, Concrete	0.44 *(0.09)*	0.31 *(0.09)*	0.60 *(0.08)*	≈ 10,600
331: Primary Metal	0.48 *(0.11)*	0.36 *(0.11)*	0.49 *(0.15)*	≈ 3,560
332: Fabricated Metal	0.50 *(0.05)*	0.59 *(0.07)*	0.57 *(0.05)*	≈ 37,700
333: Machinery	0.49 *(0.08)*	0.41 *(0.06)*	0.59 *(0.06)*	≈ 18,500
334: Computers	0.57 *(0.08)*	0.60 *(0.11)*	0.56 *(0.10)*	≈ 9,230
335: Electrical Equip	0.51 *(0.12)*	0.43 *(0.14)*	0.66 *(0.15)*	≈ 4,080
336: Transportation Equip	0.63 *(0.08)*	0.59 *(0.11)*	0.44 *(0.10)*	≈ 7,030
337: Furniture	0.25 *(0.09)*	0.30 *(0.11)*	0.55 *(0.09)*	≈ 10,300
339: Misc	0.35 *(0.07)*	0.44 *(0.09)*	0.68 *(0.05)*	≈ 12,500

Note: All regressions include 6 digit NAICS industry fixed effects, age fixed effects, and a multiunit status indicator and have standard errors clustered at the commuting zone level. Wages are based upon LBD data and as defined in the text.

Table C.15 Elasticities of Substitution between Labor and Capital for Three Digit NAICS Industries Using All Instruments

Industry	1997	2002	2007	N (1997)
311: Food Products	0.59 *(0.07)*	0.74 *(0.09)*	0.71 *(0.07)*	≈ 15,000
312: Beverage	0.87 *(0.35)*	0.86 *(0.23)*	1.2 *(0.25)*	≈ 1,380
313: Textiles	0.35 *(0.20)*	0.15 *(0.17)*	0.06 *(0.23)*	≈ 2,650
314: Textile Products	0.18 *(0.12)*	0.20 *(0.17)*	0.66 *(0.14)*	≈ 4,130
315: Apparel	0.44 *(0.09)*	0.40 *(0.16)*	0.52 *(0.16)*	≈ 10,200
316: Leather	0.60 *(0.20)*	0.51 *(0.29)*	0.84 *(0.38)*	≈ 914
321: Wood Products	0.36 *(0.12)*	-0.09 *(0.15)*	0.24 *(0.08)*	≈ 11,000
322: Paper	0.36 *(0.11)*	0.53 *(0.12)*	0.63 *(0.14)*	≈ 4,420
323: Printing	0.37 *(0.06)*	0.60 *(0.07)*	0.69 *(0.05)*	≈ 23,900
324: Petroleum Refining	0.50 *(0.23)*	0.21 *(0.27)*	0.75 *(0.22)*	≈ 1,620
325: Chemicals	0.21 *(0.12)*	0.13 *(0.13)*	0.28 *(0.14)*	≈ 8,370
326: Rubber	0.40 *(0.11)*	0.41 *(0.10)*	0.29 *(0.09)*	≈ 11,300
327: Stone, Clay, Glass, Concrete	0.29 *(0.13)*	0.32 *(0.13)*	0.58 *(0.11)*	≈ 10,600
331: Primary Metal	0.32 *(0.15)*	0.13 *(0.13)*	0.58 *(0.11)*	≈ 3,560
332: Fabricated Metal	0.46 *(0.06)*	0.57 *(0.09)*	0.52 *(0.05)*	≈ 37,700
333: Machinery	0.40 *(0.10)*	0.31 *(0.08)*	0.54 *(0.07)*	≈ 18,500
334: Computers	0.54 *(0.10)*	0.63 *(0.12)*	0.55 *(0.11)*	≈ 9,230
335: Electrical Equip	0.57 *(0.14)*	0.26 *(0.17)*	0.60 *(0.19)*	≈ 4,080
336: Transportation Equip	0.61 *(0.12)*	0.44 *(0.14)*	0.33 *(0.12)*	≈ 7,030
337: Furniture	0.11 *(0.14)*	0.16 *(0.14)*	0.52 *(0.10)*	≈ 10,300
339: Misc	0.27 *(0.09)*	0.39 *(0.11)*	0.62 *(0.06)*	≈ 12,500

Note: All regressions include 6 digit NAICS industry fixed effects, age fixed effects, and a multiunit status indicator and have standard errors clustered at the commuting zone level. Wages are based upon LBD data and as defined in the text. Instruments include amenity, Bartik, and BGS instruments together and are as defined in the text.

D Other Components for the Aggregation Framework

D.1 Estimates of Heterogeneity Index

In this section, we examine relationship between the variability in the capital share and the heterogeneity index χ. To do so, we model the capital cost to labor cost ratio $\frac{rK}{wL}$ as distributed with a lognormal distribution. We first calibrate this lognormal distribution to the US plant level data, and then examine how the capital share distribution has to change for much larger estimates of χ. We assume that all simulated plants have the same size, so the cost share weights are equal across plants.

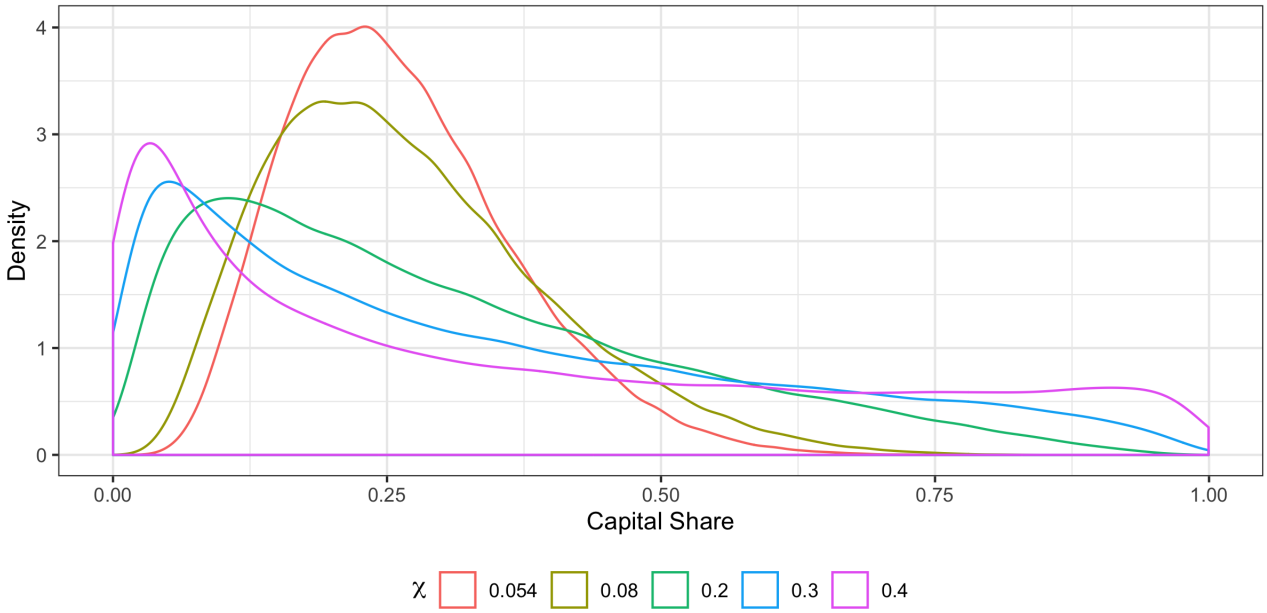
The lognormal distribution has two parameters – the mean of the logarithm μ, and the standard deviation of the logarithm σ. We calibrate μ so that the median capital share is 0.25. For σ, we use estimates of the median 75/25 ratio and 90/10 ratio of $\frac{rK}{wL}$ across 4 digit SIC industries in 1987 reported in Raval (2019), and calibrate σ to match either moment. The median 75/25 ratio in 1987 for $\frac{rK}{wL}$ is 2.1, and the median 90/10 ratio is 5.7. Using 100,000 simulation draws, we find that calibrating to the 75/25 ratio results in a χ value of 0.054, and calibrating to the 90/10 ratio results in a χ value of 0.08. These estimates are within the range of estimates of χ reported in Section 3.2, although lower than the average value of χ. However, they should be slightly smaller as they reflect differences within 4 digit industries rather than 2 digit industries, and capital shares are more disperse within 2 digit industries.

We then calibrate the lognormal distribution to match values of χ of 0.2, 0.3, and 0.4. Figure D.1 depicts the distribution of capital shares for both the χ values of 0.054 and 0.08, which match the Census data, as well as the larger χ values. In order to rationalize a larger χ value, we would need to have a much larger share of plants with extremely low and extremely high capital shares. For example, given a value of χ of 0.08, 10% of capital shares are above 0.44 and 10% of capital shares are below 12%. For a value of χ of 0.2, 10% of capital shares are above 0.6 and 10% below 0.07; for a value of χ of 0.4, 10% of capital shares are above 0.83 and 10% of capital shares are below 0.02. For a value of χ of 0.2, the 90-10 ratio for the capital cost to labor cost ratio would be 20; for a value of χ of 0.3 the 90-10 ratio would be 60; for a value of χ of 0.4 the 90-10 ratio would be 212. Thus, we would need much more dispersion in capital shares than we see in the US data in order to have values of χ substantially above what we report in Figure 1a.

D.2 Demand Elasticity Estimates

We report tables of plant demand elasticity estimates using our baseline strategy of inverting the average markup across plants in Table D.1 and Table D.2.

Figure D.1 Distribution of Capital Shares for Different Values of χ



Note: Each curve depicts the distribution of capital shares calibrated for a given value of χ, assuming that the capital cost to labor cost ratio is log normally distributed and the mean capital share is 0.25.

Table D.1 Elasticities of Demand for Two Digit SIC Industries

Industry	1987	1992	1997	N (1987)
20: Food Products	4.79 *(0.06)*	4.29 *(0.06)*	3.48 *(0.08)*	≈ 11,200
22: Textiles	7.42 *(0.21)*	5.74 *(0.17)*	5.48 *(0.13)*	≈ 3,580
23: Apparel	3.94 *(0.04)*	3.67 *(0.05)*	3.75 *(0.04)*	≈ 12,800
24: Lumber and Wood	6.60 *(0.12)*	4.99 *(0.08)*	5.52 *(0.05)*	≈ 15,500
25: Furniture	4.48 *(0.05)*	4.07 *(0.08)*	4.18 *(0.06)*	≈ 5,720
26: Paper	6.96 *(0.13)*	5.81 *(0.09)*	5.11 *(0.08)*	≈ 4,280
27: Printing and Publishing	3.44 *(0.02)*	3.14 *(0.02)*	3.91 *(0.03)*	≈ 27,800
28: Chemicals	3.58 *(0.06)*	3.21 *(0.04)*	2.93 *(0.04)*	≈ 7,040
29: Petroleum Refining	5.62 *(0.23)*	5.55 *(0.20)*	3.60 *(1.05)*	≈ 1,670
30: Rubber	5.39 *(0.07)*	4.33 *(0.05)*	4.16 *(0.04)*	≈ 8,630
31: Leather	4.28 *(0.12)*	3.84 *(0.14)*	3.72 *(0.12)*	≈ 1,000
32: Stone, Clay, Glass, Concrete	6.16 *(0.09)*	5.35 *(0.07)*	4.09 *(0.06)*	≈ 9,360
33: Primary Metal	7.32 *(0.21)*	5.44 *(0.13)*	4.04 *(0.07)*	≈ 4,380
34: Fabricated Metal	4.79 *(0.04)*	4.37 *(0.04)*	3.85 *(0.03)*	≈ 21,000
35: Machinery	4.24 *(0.03)*	4.08 *(0.07)*	3.96 *(0.02)*	≈ 26,100
36: Electrical Machinery	3.89 *(0.04)*	3.65 *(0.03)*	3.45 *(0.04)*	≈ 8,300
37: Transportation Equip	5.12 *(0.18)*	4.87 *(0.09)*	4.55 *(0.07)*	≈ 5,130
38: Instruments	3.20 *(0.04)*	2.94 *(0.03)*	3.00 *(0.03)*	≈ 4,680
39: Misc	4.08 *(0.04)*	3.68 *(0.04)*	3.48 *(0.04)*	≈ 6,900

Note: All estimates are based upon inverting the average markup across plants in an industry; the markup over marginal cost is equal to $\frac{\varepsilon}{\varepsilon-1}$. We define the markup as sales divided by the sum of costs from capital, labor, and materials.

Table D.2 Elasticities of Demand for Three Digit NAICS Industries

Industry	1997	2002	2007	N (1997)
311: Food Products	3.52 *(0.05)*	3.06 *(0.04)*	2.76 *(0.04)*	≈ 15,000
312: Beverage	3.10 *(0.32)*	2.79 *(0.07)*	2.68 *(0.06)*	≈ 1,380
313: Textiles	5.42 *(0.15)*	5.86 *(0.21)*	4.40 *(0.18)*	≈ 2,650
314: Textile Products	4.52 *(0.09)*	4.30 *(0.09)*	3.50 *(0.06)*	≈ 4,130
315: Apparel	3.58 *(0.05)*	3.46 *(0.06)*	3.02 *(0.04)*	≈ 10,200
316: Leather	3.69 *(0.12)*	3.64 *(0.12)*	3.85 *(0.14)*	≈ 914
321: Wood Products	5.93 *(0.06)*	5.00 *(0.08)*	4.54 *(0.08)*	≈ 11,000
322: Paper	5.06 *(0.08)*	4.85 *(0.08)*	4.22 *(0.09)*	≈ 4,420
323: Printing	3.92 *(0.03)*	3.48 *(0.02)*	3.45 *(0.03)*	≈ 23,900
324: Petroleum Refining	3.60 *(1.05)*	4.35 *(0.19)*	3.22 *(0.17)*	≈ 1,620
325: Chemicals	2.96 *(0.04)*	2.79 *(0.04)*	2.55 *(0.04)*	≈ 8,370
326: Rubber	4.28 *(0.04)*	3.99 *(0.04)*	3.86 *(0.04)*	≈ 11,300
327: Stone, Clay, Glass, Concrete	4.10 *(0.06)*	3.56 *(0.04)*	3.08 *(0.04)*	≈ 10,600
331: Primary Metal	4.28 *(0.09)*	4.29 *(0.11)*	3.33 *(0.16)*	≈ 3,560
332: Fabricated Metal	3.83 *(0.02)*	3.61 *(0.02)*	3.14 *(0.02)*	≈ 37,700
333: Machinery	4.00 *(0.03)*	3.84 *(0.03)*	3.50 *(0.03)*	≈ 18,500
334: Computers	3.39 *(0.03)*	3.59 *(0.04)*	3.07 *(0.03)*	≈ 9,230
335: Electrical Equip	3.36 *(0.07)*	3.41 *(0.06)*	3.14 *(0.05)*	≈ 4,080
336: Transportation Equip	4.60 *(0.07)*	4.29 *(0.07)*	4.09 *(0.07)*	≈ 7,030
337: Furniture	4.44 *(0.05)*	3.75 *(0.03)*	3.56 *(0.03)*	≈ 10,300
339: Misc	3.13 *(0.02)*	3.22 *(0.02)*	3.03 *(0.02)*	≈ 12,500

Note: All estimates are based upon inverting the average markup across plants in an industry; the markup over marginal cost is equal to $\frac{\varepsilon}{\varepsilon-1}$. We define the markup as sales divided by the sum of costs from capital, labor, and materials.

D.3 Elasticity of Demand and Returns to Scale

Our baseline strategy to estimate the demand elasticity is to invert the average markup. For several homogeneous products, the US Census of Manufactures collects both price and physical quantity data. For these industries, we can use an approach similar to Foster et al. (2008) and estimate the elasticity of demand by regressing quantity on price, instrumenting for price using average cost. Given these demand estimates, the average industry-level capital-labor elasticity of substitution is 0.44 among these industries, close to the estimate of 0.50 using our baseline strategy for the same industries.[9] See Web Appendix B.5 for details of the data construction. The trade literature finds estimates in the same range as our baseline estimates using within industry variation across imported varieties to identify the elasticity of demand. For example, Imbs and Mejean (2015) find a median elasticity of 4.1 across manufacturing industries.

In an environment with arbitrary demand elasticities and imperfect pass-through of marginal cost, the formula for the industry elasticity in Proposition 1 is unchanged except the elasticity of demand ε_n is replaced by a weighted average of the quantity $b_{ni}\varepsilon_{ni}$; ε_{ni} is i's local demand elasticity and b_{ni} is i's local rate of relative pass-through (the elasticity of its price to a change in marginal cost). We show this in Web Appendix G.2.3 where we only restrict the demand system to be homothetic. Under Dixit-Stiglitz preferences, $b_{ni} = 1$ and $\varepsilon_{ni} = \varepsilon_n$ for each i. Here, however, if a plant passes through only three-quarters of a marginal cost increase, then the subsequent change in scale would be three-quarters as large. Given a pass-through rate of three-quarters, our estimate of the 1987 aggregate elasticity of substitution would be 0.66, compared to 0.72 at baseline.

Our baseline estimation assumed that each plant produced using a production function with constant returns to scale. Alternatively, we can assume that plant i produces using the production function:

$$Y_{ni} = F_{ni}(K_{ni}, L_{ni}, M_{ni}) = G_{ni}(K_{ni}, L_{ni}, M_{ni})^{\gamma}$$

where G_{ni} has constant returns to scale and $\gamma < \frac{\varepsilon_n}{\varepsilon_n - 1}$. Relative to the baseline, two things change, as shown in Web Appendix G.2.2. First, the industry elasticity of substitution becomes

$$\sigma_n^N = (1 - \chi_n)\bar{\sigma}_n + \chi_n \left[\bar{s}_n^M \bar{\zeta}_n + (1 - \bar{s}_n^M) x_n\right]$$

where x_n is defined to satisfy $\frac{x_n}{x_n - 1} = \frac{1}{\gamma}\frac{\varepsilon_n}{\varepsilon_n - 1}$. Thus the scale elasticity is a composite of two parameters, the elasticity of demand and the returns to scale. When the wage falls, the amount a labor-intensive plant would expand depends on both.

Second, when we divide a plant's revenue by total cost, we no longer recover the markup. Instead, we get

$$\frac{P_{ni}Y_{ni}}{rK_{ni} + wL_{ni} + qM_{ni}} = \frac{1}{\gamma}\frac{\varepsilon_n}{\varepsilon_n - 1} = \frac{x_n}{x_n - 1}$$

Fortunately, this means that the procedure we used in the baseline delivers the correct aggregate elasticity of substitution even if we mis-specify the returns to scale. To see this, when we assumed constant returns to scale, we found the elasticity of demand by computing $\frac{P_{ni}Y_{ni}}{P_{ni}Y_{ni} - (rK_{ni} + wL_{ni} + qM_{ni})}$. With alternative returns to scale, this would no longer give the elasticity of demand, ε_n; rather, it gives the correct scale elasticity, x_n.

[9] Foster et al. (2008) instrument for price using plant-level TFP. We cannot use their estimates directly because they assume plants produce using homogeneous Cobb-Douglas production functions. Because we maintain the assumption of constant returns to scale, the appropriate analogue to plant-level TFP is average cost. Directly using the demand elasticities of Foster et al. (2008) would yield an average industry-level elasticity of substitution of 0.46.

D.4 Local Content of Materials

In our baseline estimates of the elasticity of substitution between materials and non-materials, ζ, we assume that the local wage does not affect the materials price the plant faces. As a robustness check, we examine how sensitive our estimates are to correlation between materials prices and local wages due to local content of materials. The local wage would affect labor costs for locally sourced materials. We use the 1993 Commodity Flow Survey to construct the local content of shipments for every industry included in the survey, defining local as a shipment within 100 miles of the originating factory. We then use the 1992 Input-Output tables to construct the average local content of materials for every manufacturing industry. Assuming that every input industry has the same materials and labor shares and fraction of local content of materials, the elasticity of the materials price with respect to the wage is:

$$\frac{d\log q_i}{d\log w} = (1-\alpha_n)\frac{1-s_n^M l_n}{1-(1-s_n^M)l_n}$$

where l_n is the measure of local content for industry n.

We therefore estimate ζ using the regression

$$\log\frac{rK_{nic}+wL_{nic}}{qM_{nic}} = (1-\zeta)\frac{(1-\alpha_{nic})}{(1-\alpha_n)\frac{1-s_n^M l}{1-(1-s_n^M)l_n}}(\log w_c)+\gamma X_{nic}+\epsilon_{nic}$$

We find only slightly lower estimates in estimated elasticities after accounting for the local content of materials. Our estimate of this elasticity accounting for local content of materials are 1.01 in 1987 and 0.80 in 1992 using worker based wages, compared to 1.03 in 1987 and 0.83 in 1992 under our baseline estimates.

D.5 Cross Industry Demand Elasticity

The cross industry elasticity of demand characterizes how industry-level demand responds to a change in the overall industry price level. To estimate this elasticity, we use panel data on quantity and price at the industry level from the NBER productivity database from 1962 to 2009.

Since least squares estimates conflate demand and supply, we have to instrument for price using supply side instruments that capture industry productivity. The two instruments that we examine are the average product of labor, defined as the amount of output produced per worker, and the average real cost per unit of output produced, which is the appropriate measure of industry productivity in our model. We thus have the following regression specification:

$$\log q_{n,t} = -\eta\log p_{n,t}+\alpha_n+\beta_t+CONTROLS+\varepsilon_n$$

where $q_{n,t}$ is quantity produced for industry n in period t, $p_{n,t}$ is the price for industry n in period t, α_n are a set of industry fixed effects, and β_t are a set of time fixed effects.

We then examine the cross industry demand elasticity, defining industry at both the four digit and two digit SIC levels. We have 459 four digit industries and 20 two digit industries.[10] For each

[10] Since the underlying data is at the four digit industry level, we develop two digit SIC prices and quantities using a Fisher ideal index with base year 1987. We also exclude eight 4 digit industries which disappear because they are excluded after the Census shifts to NAICS basis manufacturing, the most prominent of which is Newspaper Publishing.

industry definition, we develop specifications with extra sets of controls to account for potential trends over time that could be correlated with changes in prices. In the four digit specifications, these extra controls include either 2 digit industry-year fixed effects, or 4 digit industry linear trends. In the two digit specifications, these extra controls include 2 digit industry linear trends.

Table D.3 below contains these estimates, as well as the OLS estimate. As would be expected from simultaneity bias, OLS estimates are lower in magnitude than IV estimates. The IV estimates using four digit industries range between 1.2 and 2.2 and are slightly above estimates using two digit industries. This pattern is consistent with two digit industry varieties being less substitutable than four digit industry varieties.

The two digit industry IV estimates range from 0.75 to 1.15, with three of the four estimates close to one. Because we define industries in our aggregation analysis at the two digit level, the two digit industry estimates are more appropriate. We thus set the cross industry demand elasticity to one. Our results are not extremely sensitive to this elasticity; increasing the elasticity from 1 to 1.5 would increase the US aggregate elasticity by about 0.01.

Table D.3 Cross Industry Elasticity of Demand for the Manufacturing Sector

	Industry Definition:				
Instrument	Four Digit			Two Digit	
None	0.99	1.06	0.57	0.91	0.37
	(0.02)	*(0.01)*	*(0.02)*	*(0.03)*	*(0.05)*
APL	1.30	1.28	2.12	1.14	1.05
	(0.01)	*(0.01)*	*(0.03)*	*(0.04)*	*(0.06)*
Avg Cost	1.19	1.22	1.58	1.04	0.77
	(0.01)	*(0.01)*	*(0.02)*	*(0.03)*	*(0.05)*
Industry-Year Controls	None	Two Digit FE	Four Digit Trends	None	Two Digit Trends

Note: Standard errors are in parentheses. The first row contains coefficients from OLS regressions, while the second and third row are IV regressions with either the average product of labor or average real cost per unit produced as instruments. The first three columns are on four digit SIC industries; all regressions contain four digit SIC industry and year fixed effects. The second column also includes two digit industry-year fixed effects and the third column also includes four digit industry linear time trends. The last two columns are on two digit SIC industries; all regressions contain two digit SIC industry and year fixed effects. The last column also includes two digit industry linear time trends.

E Aggregation

E.1 Aggregate US Elasticity

In the body of the paper, we restrict our analysis to the manufacturing sector because, for the US, we only have micro data on capital and labor for manufacturing. However, in this section, we demonstrate how our approach could be used to estimate the aggregate US elasticity. We apply our framework to an economy with two sectors – manufacturing and services. The two papers that estimate the capital-labor substitution elasticity for services – Alvarez-Cuadrado et al. (2018) and Herrendorf et al. (2015) – do so in a value added framework, so we apply our framework without materials. In that case, we need to know three elasticities - the capital-labor substitution elasticities for services and manufacturing, and the elasticity between the services and manufacturing sectors.

Alvarez-Cuadrado et al. (2018) calibrate their model using a services-manufacturing elasticity of either 0 or 0.5, while Herrendorf et al. (2013), a companion paper to Herrendorf et al. (2015), estimate a services-manufacturing elasticity close to zero under a sectoral value added framework. Thus, we examine the aggregate elasticity using the estimates from both papers and a services-manufacturing elasticity of 0 and 0.5. We use 1997 KLEMS data provided by Dale Jorgenson (Jorgenson, 2008) to construct value added shares and the cross-sector capital share variance.

Table E.1 contains the estimates of the aggregate elasticity under these assumptions. Because the differences in capital shares between manufacturing and services are quite small, our estimates are very insensitive to the cross sector elasticity. The capital share for manufacturing is 0.35, compared to 0.36 for services, so the cross industry weight is 0.0001. The aggregate elasticity is thus very close to the average sector elasticity weighting sectors by value added. This approach results in an aggregate elasticity of 0.62 using the estimates of Alvarez-Cuadrado et al. (2018), and 0.76 using the estimates of Herrendorf et al. (2015). Since both papers estimate sector level elasticities less than one for manufacturing and services, the aggregate elasticity remains less than one.

Table E.1 Estimates of the Aggregate US Elasticity of Substitution

Paper	Manufacturing Elasticity	Services Elasticity	Cross Sector Elasticity	Aggregate Elasticity
Alvarez-Cuadrado et al. (2018)	0.77	0.57	0	0.62
Alvarez-Cuadrado et al. (2018)	0.88	0.53	0.5	0.62
Herrendorf et al. (2015)	0.8	0.75	0	0.76
Herrendorf et al. (2015)	0.8	0.75	0.5	0.76

Note: We take the capital-labor manufacturing and services elasticities from the respective papers, and construct the aggregate elasticity using sector level data on capital and labor costs from Jorgenson (2008) with the same definitions of manufacturing and services as Alvarez-Cuadrado et al. (2018), and use 1997 data to estimate the cross-industry weight and value added shares.

E.2 Aggregation of Micro Data

We now compare our methodology to an approach that would aggregate the data to industry or sector level for each local area, and then estimate the elasticity using cross-sectional variation on the aggregated data. This approach was used in the past to estimate the capital-labor elasticity (Lucas, 1969). However, as Lucas (1969) points out, the old cross-sectional literature suffered from a number of problems, including differences in industry composition across areas.

We thus examine aggregation at three different levels across commuting zones: at the manufacturing sector level, at the 2 digit SIC / 3 digit NAICS level, and at the 4 digit SIC / 6 digit NAICS level. Because both commuting zones and industries vary substantially in their degree of economic activity, we examine specifications that weight the data by value added as well as unweighted specifications.

Table E.2 contains the results of these specifications. At the manufacturing and 2/3 digit industry level, the estimates vary substantially by weighting procedure. The estimates are above one for the manufacturing sector level unweighted, while they are negative or close to zero after weighting using value added. At the 2/3 digit level, the estimates remain much higher in the unweighted specifications than the weighted specifications, although the difference is not as stark as at the manufacturing sector level. In both cases, the unweighted elasticities are substantially higher than our baseline industry level estimates, contained in the second column of Table E.2, while the weighted elasticities are substantially lower than our baseline industry level estimates.

At the 4 or 6 digit industry level, however, estimates of the elasticity of substitution for the unweighted and weighted estimates are fairly similar both to each other and to our baseline industry level estimates. Our baseline industry level estimates range between 0.5 and 0.7 across years, compared to between 0.45 and 0.7 across years for the estimates aggregating to the 4/6 digit industry level. Thus, differences in industry composition likely biased the more aggregated estimates. In addition, the fact that the estimates aggregated to the 4/6 digit level matched the estimates from our aggregation framework provides confidence in our theory of aggregation.

Table E.2 Cross-Sectional Aggregation Estimates of the Capital-Labor Substitution Elasticity

	None	Manufacturing		2/3 Digit		4/6 Digit	
1987	0.68	1.27 *(0.20)*	-0.45 *(0.29)*	0.94 *(0.07)*	0.21 *(0.17)*	0.60 *(0.04)*	0.64 *(0.07)*
1992	0.67	1.62 *(0.21)*	-0.38 *(0.27)*	1.16 *(0.07)*	0.22 *(0.16)*	0.70 *(0.04)*	0.54 *(0.08)*
1997	0.51	1.23 *(0.23)*	-0.08 *(0.23)*	1.04 *(0.08)*	0.22 *(0.17)*	0.63 *(0.05)*	0.59 *(0.10)*
2002	0.52	1.40 *(0.22)*	0.01 *(0.18)*	0.93 *(0.09)*	0.14 *(0.16)*	0.68 *(0.06)*	0.69 *(0.04)*
2007	0.53	1.08 *(0.21)*	0.00 *(0.21)*	0.63 *(0.07)*	0.04 *(0.14)*	0.46 *(0.04)*	0.45 *(0.10)*
Weight	NA	None	Value Added	None	Value Added	None	Value Added

Note: Standard errors are in parentheses. The table contains seven specifications. The second column is the average industry level elasticity using cost share weights based upon our aggregation framework, and allowing all elasticities to vary by year. The third and fourth columns estimate the elasticity of substitution after aggregating the data to the manufacturing sector level. The fifth and sixth columns estimate the elasticity of substitution after aggregating the data to the 2 digit SIC or 3 digit NAICS industry level. The sixth and seventh columns estimate the elasticity of substitution after aggregating the data to the 4 digit SIC or 6 digit NAICS industry level. For each level of aggregation, we either estimate the specification unweighted or weighted by total value added.

All regressions include industry fixed effects, where applicable. Wages used are the average log wage for the commuting zone, computed as wage and salary income over total number of hours worked adjusted for differences in worker characteristics. Standard errors are clustered at the commuting zone level.

E.3 Aggregate Time Series Approaches

We now compare our methodology to the approach that jointly estimates the aggregate capital-labor elasticity of substitution and bias of technical change using aggregate time series data. This approach uses the following econometric model:

$$\frac{s^{v,L}}{1-s^{v,L}} = \beta_0 + (\sigma^{agg} - 1)\log\frac{r}{w} + \log\phi + \epsilon \tag{E.1}$$

where $d\log\phi$ is the bias of technical change and ϵ is interpreted as measurement error that is orthogonal to $\log\frac{r}{w}$. It is well known that estimates depend critically on what assumptions are placed on the bias of technical change. We examine different assumptions on the bias of technical change on data aggregated to the manufacturing sector level for 1970 to 2010 from the NBER Productivity Database.

Under an assumption of Hicks neutral technical change ($d\log\phi = 0$), the aggregate elasticity is precisely estimated at 1.91. The elasticity is considerably above one because the labor share fell and wages rose relative to capital prices during the sample period.

Once we allow for biased technical change, however, estimates of both the bias and aggregate elasticity become imprecise, as shown in Figure E.1. The first way we introduce biased technical change is through a constant rate of biased technical change ($d\log\phi$ is constant). This constant rate of bias becomes a time trend in the aggregate regression. The elasticity is then identified by movements in relative factor prices around the trend; short run movements in factor prices are assumed to be uncorrelated with movements in technology. Given a constant bias, the estimate of the aggregate elasticity using least squares regressions is 0.56; the 95 percent confidence interval ranges from 0.05 to 1.07.

Our evidence for a rising rate of biased technical change over time motivates the use of a more flexible specification for the bias. We use a Box–Cox transformation of the time trend, as in Klump et al. (2007), which allows the bias to vary monotonically over time.[11] With the Box–Cox specification, the aggregate elasticity is 0.69, close to our baseline estimates. Again, the range of the confidence interval is large.

Each methodology provides a measure of the contribution of the bias of technical change to the decline in the labor share, depicted in Figure E.1.[12] Assuming a constant rate of biased technical change, the average contribution of bias is about −0.5 percentage points per year and is larger than our average contribution. More importantly, this average misses the timing of the large changes in the contribution of bias over time. The Box-Cox specification implies that the contribution of bias to the labor share has accelerated over time, but does not display the sharp drop at 2000 that the bias estimates from our method have.

E.4 Capital Prices and the Cross-Country Approach

Karabarbounis and Neiman (2014) take an alternative approach to estimating a long-run aggregate elasticity of capital-labor substitution by studying how long-run changes in factor prices alter capital shares for a panel of countries. Using this approach, they find a country-level elasticity of substitution of 1.25, which is notably larger than one. This estimate is not directly comparable to ours because the data differ in terms of sectoral coverage, countries, and time period. Nevertheless, we believe it would be instructive to compare the two approaches.

One can recover the capital-labor elasticity of substitution σ by estimating multiple different first order conditions. Consider the three regressions that use different combinations of FOCs below:

$$\ln\frac{rK}{wL} = \alpha_0 + (\sigma-1)\ln\frac{w}{r} + (\sigma-1)\ln\frac{A}{B} \tag{E.2}$$

$$\ln\frac{rK}{PY} = \alpha_0 + (1-\sigma)\ln\frac{r}{P} + (\sigma-1)\ln A \tag{E.3}$$

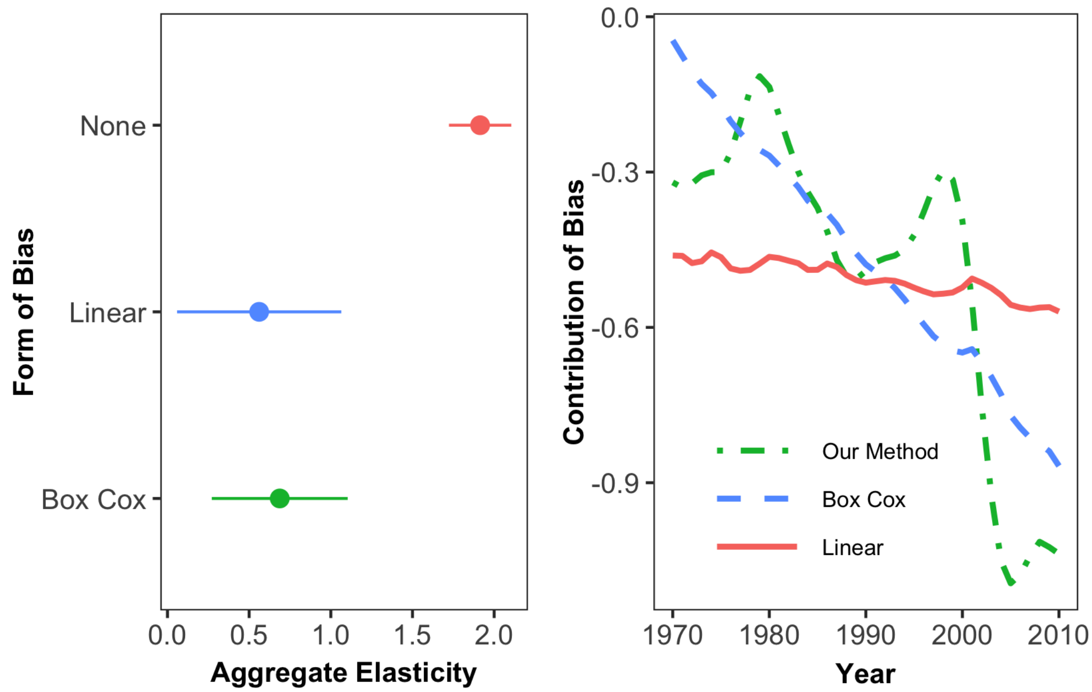
$$\ln\frac{wL}{PY} = \alpha_0 + (1-\sigma)\ln\frac{w}{P} + (\sigma-1)\ln B \tag{E.4}$$

If the technology term is treated as a residual, using OLS to estimate σ requires respectively three different orthogonality assumptions: $\ln\frac{w}{r} \perp \ln\frac{A}{B}$, $\ln r \perp \ln A$, and $\ln w \perp \ln B$.

[11] The Box–Cox transformation implies that $d\log\phi = \gamma t^{\lambda}$; λ allows the rate of biased technical change to vary over time.

[12] For the aggregate time series method, the contribution of bias is $s^{v,L}(1-s^{v,L})\mathrm{d}\log\phi$; thus, the contribution to the labor share can vary over time even if the bias is constant.

Figure E.1 Elasticity and Bias Estimates from Aggregate Data



Note: The left plot displays the point estimate and 95 percent confidence interval for the aggregate elasticity of substitution from regressions based on (E.1). Specifications differ in assumptions on the bias of technical change. Technical change is respectively assumed to have no trend, follow a linear time trend, or follow a Box–Cox transformation of the time trend. The right plot displays the contribution to the labor share from the bias of technical change, from either aggregate regressions with a linear or Box-Cox specification of the time trend or from our method that estimates the aggregate elasticity from the micro data.

Karabarbounis and Neiman (2014) estimate the elasticity using changes in the representative firm's first order condition for capital (E.3), using cross-country variation in capital price changes. They examine two different identifying assumptions. First, they assume that technology is purely labor-augmenting, i.e. $d\ln A = 0$, arguing that this would be required on a balanced growth path. Second, they assume that technical change is purely Hicks neutral, in which case they can measure $d\ln A$ directly because it is equal to the change in TFP.

Relative to the approaches that estimate aggregate elasticities using the aggregate time series, Karabarbounis and Neiman (2014) take a step forward by using a panel of countries and focusing on long-run variation. However, we believe that the assumptions that technical change has been either purely labor-augmenting or purely Hicks-neutral are overly restrictive, especially in a time when factor shares have been changing. This is especially clear once one recognizes that changes in the capital- and labor-augmenting productivity residuals (A and B) capture many things that are not purely technological in nature such as shifts in composition due to structural change or offshoring, or from a myriad of other changes. For example, as Mutreja et al. (2018) show, most countries import almost all of their capital goods, so much of the variation within countries may follow trade liberalizations. However, trade liberalizations often occur alongside other reforms. Changes in trade barriers or these other reforms can affect capital shares for multiple reasons other than simply shifting capital prices, which would bias any estimates of the elasticity.

Estimates of the elasticity using OLS can differ when using each of the three FOCs. A natural interpretation is that factor prices are correlated with technology, and the different regressions require different orthogonality restrictions. In contrast, if one has an instrument for factor prices that is orthogonal to A and B, one can sidestep these issues completely. We use the combined labor and capital first order condition (E.2) because we have variation in wages in the cross-section across locations that is plausibly orthogonal to the bias of technology. An alternative approach to estimate the elasticity would be to use differences in the cost of capital across firms or industries. Multiple papers that use plausibly exogenous variation in the cost of capital, and the capital first order condition, find similar estimates of the micro elasticity to our work using wage differences and the labor and capital first order conditions. We summarize these results in Section 4.1.

F Labor Share Decomposition

F.1 Discrete Approximation

This section shows the discrete approximation we use to approximate (16). For any x, define

$$\begin{aligned} \bar{x} &= \frac{x_{t+1}+x_t}{2} \\ \Delta x &= \frac{x_{t+1}-x_t}{\bar{x}} \end{aligned}$$

This means that for any x, y, we have

$$\begin{aligned} \Delta\left(x+y\right) &= \frac{x_{t+1}+y_{t+1}-x_t-y_t}{\bar{x}+\bar{y}} = \frac{\bar{x}}{\bar{x}+\bar{y}}\Delta x + \frac{\bar{y}}{\bar{x}+\bar{y}}\Delta y \\ \Delta\left(x-y\right) &= \frac{x_{t+1}-y_{t+1}-x_t+y_t}{\bar{x}-\bar{y}} = \frac{\bar{x}}{\bar{x}-\bar{y}}\Delta x - \frac{\bar{y}}{\bar{x}-\bar{y}}\Delta y \end{aligned}$$

Two useful relationships are:

$$\begin{aligned} \Delta\left(s^{v,K}+s^{v,L}\right) &= \frac{\bar{s}^{v,K}}{\bar{s}^{v,K}+\bar{s}^{v,L}}\Delta s^{v,K} + \frac{\bar{s}^{v,L}}{\bar{s}^{v,K}+\bar{s}^{v,L}}\Delta s^{v,L} \\ \left(1-\bar{s}^{v,\pi}\right)\Delta\left(1-s^{v,\pi}\right) &= -\bar{s}^{v,\pi}\Delta s^{v,\pi} \end{aligned}$$

Define biases of technical change

$$\begin{aligned} \phi_{kl} &\equiv \Delta rK - \Delta wL - (1-\sigma^{agg})\Delta\frac{r}{w} \\ \phi_{\pi} &\equiv \Delta s^{v,\pi} - \Delta\left(1-s^{v,\pi}\right) \end{aligned}$$

These can be written as

$$\begin{aligned} \phi_{k,l} &= \Delta s^{v,K} - \Delta s^{v,L} - (1-\sigma^{agg})\Delta\frac{r}{w} \\ \phi_{\pi} &= -\frac{1}{\bar{s}^{v,\pi}}\Delta\left(1-s^{v,\pi}\right) \end{aligned}$$

The labor share can then be decomposed as

$$\begin{aligned}\Delta s^{v,L} &= \Delta s^{v,L} - \Delta\left(s^{v,K} + s^{v,L}\right) + \Delta\left(1 - s^{v,\pi}\right) \\ &= \Delta s^{v,L} - \frac{\bar{s}^{v,K}}{\bar{s}^{v,k} + \bar{s}^{v,L}}\Delta s^{v,K} - \frac{\bar{s}^{v,L}}{\bar{s}^{v,K} + \bar{s}^{v,L}}\Delta s^{v,L} + \Delta\left(1 - s^{v,\pi}\right) \\ &= \frac{\bar{s}^{v,K}}{\bar{s}^{v,K} + \bar{s}^{v,L}}\left(\Delta s^{v,L} - \Delta s^{v,K}\right) + \Delta\left(1 - s^{v,\pi}\right)\end{aligned}$$

Using the definitions of $\phi_{k,l}$ and ϕ_π we have

$$\Delta s^{v,L} = \frac{\bar{s}^{v,K}}{\bar{s}^{v,K} + \bar{s}^{v,L}}(\sigma^{agg} - 1)\Delta\frac{r}{w} - \frac{\bar{s}^{v,K}}{\bar{s}^{v,K} + \bar{s}^{v,L}}\phi_{k,l} - \bar{s}^{v,\pi}\phi_\pi$$

In the decomposition, we then use

$$s^{v,L}_{t+1} - s^{v,L}_t = \bar{s}^{v,L}\left[\frac{\bar{s}^{v,K}}{\bar{s}^{v,K} + \bar{s}^{v,L}}(\sigma^{agg} - 1)\Delta\frac{r}{w} - \frac{\bar{s}^{v,K}}{\bar{s}^{v,K} + \bar{s}^{v,L}}\phi_{k,l} - \bar{s}^{v,\pi}\phi_\pi\right] \tag{F.1}$$

F.2 Profit Share and Unmeasured Payments to Capital

In our benchmark decomposition, we assume that factor prices do not affect $s^{v,\pi}$. This assumption implicitly views changes in $s^{v,\pi}$ as stemming from shifts in preferences that change markups or from changes in the share of materials. The top panel of Table F.1 contains these estimates.

We can decompose the total contribution of bias into a contribution from capital/labor bias and a contribution from profit bias. From 1970-1999, the contribution of capital/labor bias was -0.25 percentage points per year (-7.6 cumulative), while that of profit bias was -0.10 percentage points per year (-3.0 cumulative). From 2000-2010, the contribution from capital/labor bias was -0.37 percentage points per year (-4.1 cumulative), while that of profit bias was -0.50 percentage points per year (-5.5 cumulative). Thus, while contributions from both the capital-labor bias and profit bias accelerated in the 2000-2010 period, the profit bias contributes more to the post-2000 acceleration in the total contribution.

In this section, we also examine an alternative view put forward by Karabarbounis and Neiman (2019): that $s^{v,\pi}$ reflects unmeasured payments to capital. This could happen if either the measured capital stock understates the quantity of capital (e.g., it misses intangible capital) or the measured rental rate understates the required return to capital. In either case, what we had labeled as $s^{v,\pi}$ would actually respond to factor prices as well. To assess how this alternative view would affect our conclusions, we perform the decomposition under the assumption that all of $s^{v,\pi}$ reflects unmeasured payments to capital. To do this, we use (F.1) but set $s^{v,K} = 1 - s^{v,L}$ and $s^{v,\pi} = 0$.

$$s^{v,L}_{t+1} - s^{v,L}_t = \bar{s}^{v,L}(1 - \bar{s}^{v,L})(\sigma^{agg} - 1)\Delta\frac{r}{w} - \bar{s}^{v,L}(1 - \bar{s}^{v,L})\phi_{k,l} \tag{F.2}$$

The bottom panel of Table F.1 contains these estimates. Compared to our baseline, the contribution from factor prices is higher in magnitude, at 0.18 percentage points per year from 1970-1999 and 0.14 percentage points per year from 1970-2010. Thus, the contribution of the bias is larger, with a decline of 0.43 percentage points per year from 1970 to 1999 (-13.0 cumulative), and 0.93 percentage points per year from 2000 to 2010 (-10.2 cumulative). In total, the contribution of the bias is 3 percentage points higher than our baseline estimate.

Table F.1 Contributions to Labor Share Change: Unmeasured Payments to Capital

	Annual Contribution			Cumulative Contribution		
Period	Labor Share	Factor Prices	Bias	Labor Share	Factor Prices	Bias
			Profit Share			
1970-1999	-0.25	0.10	-0.35	-7.61	3.00	-10.62
2000-2010	-0.79	0.09	-0.88	-8.73	0.94	-9.66
			Unmeasured Payments to Capital			
1970-1999	-0.25	0.18	-0.43	-7.61	5.41	-13.02
2000-2010	-0.79	0.14	-0.93	-8.73	1.51	-10.24

Note: The factor price and bias contributions are as defined in the text. Annual Contributions are in percentage points per year and Cumulative Contributions are in percentage points.

F.3 Alternative Rental Prices

Our rental prices are based upon official NIPA deflators for equipment and structures capital. We develop a Tornqvist index for the rental price to account for changing shares of two digit industries and different types of capital over time. For wages, we use BLS data on total compensation and hours for each industry, correcting for labor quality using indices from Jorgenson et al. (2013). Jorgenson et al. (2013) measures labor quality as the deviation of total hours from a Tornqvist index of hours across many different cells that represent workers with different amounts of human capital, as in Jorgenson et al. (2005).

However, Gordon (1990) has argued that the NIPA deflators underestimate the actual fall in equipment prices over time. We examine how this critique might change our results on the bias of technical change by using an alternative rental price series for equipment capital that Cummins and Violante (2002) developed by extending the work of Gordon (1990). Their series extends to 1999, so we compare our baseline to these rental prices during the 1970-1999 period. Using the Cummins and Violante (2002) equipment prices implies that the wage to rental price ratio has increased by 3.8 percent per year, instead of 2.0 percent per year with the NIPA deflators. This change increases the contribution of factor prices to the labor share from 0.10 percentage points per year to 0.18 percentage points per year, or about 2.4 percentage points over the 1970-1999 period. The contribution from the bias thus also rises by 2.4 percentage points. Given our estimate of the aggregate elasticity of substitution, changes in factor prices have not been the driving force behind the declining labor share.

Table F.2 Contributions to Labor Share Change with Alternative Rental Price Series

		Annual Contribution			Cumulative Contribution		
Deflator	Annual $\frac{w}{r}$ Change	Labor Share	Factor Prices	Bias	Labor Share	Factor Prices	Bias
NIPA	2.02	-0.25	0.10	-0.36	-7.61	3.08	-10.70
GCV	3.80	-0.25	0.18	-0.44	-7.61	5.47	-13.08

Note: The factor price and bias contributions are as defined in the text. Annual Contributions are in percentage points per year and Cumulative Contributions are in percentage points. Data covers 1970-1999.

In addition, our baseline rental price series estimates the rental price assuming a constant external real rate of return of 3.5%. This assumption could be violated if the real rate of return has changed over time. We thus also examine the external nominal rate of return specification of Harper et al. (1989). The rental rate for industry n is then defined as:

$$R_{i,t} = T_{i,t}(p_{i,t-1}r^n_{i,t} + \delta_{i,t}p_{i,t} + (p_{i,t} - p_{i,t-1}))$$

where $r^n_{i,t}$ is the Moody's BAA bond rate for the year (obtained from FRED), $p_{i,t}$ is the price index for capital in that industry, $\delta_{i,t}$ is the depreciation rate for that industry, and $T_{i,t}$ is the effective rate of capital taxation. Thus, instead of using a constant external rate of return, we use the BAA nominal rate and the realized rate of capital inflation $(p_{i,t} - p_{i,t-1})$. This specification thus substitutes the realized rate of capital inflation for the expected rate of capital inflation.

Using the nominal external rate specification implies that the wage to rental price ratio has increased by 1.1 percent per year from 1970 to 1999 and -0.14 percent per year from 2000 to 2010, instead of 2.0 and 1.4 percent per year with the external real rate specification. This change decreases the contribution of factor prices to the labor share from 0.10 percentage points per year from 1970 to 1999 and 0.09 percentage points per year from 2000 to 2010 to 0.04 and -0.02 percentage points per year. Given our estimate of the aggregate elasticity of substitution, these changes remain small relative to the patterns of decline in the labor share.

In addition, our estimates of the bias of technical change decrease slightly to about 17.5 percentage points in aggregate from 1970 to 2010.

Table F.3 Contributions to Labor Share Change with Alternative Rate of Return Series

			Annual Contribution			Cumulative Contribution		
Rate	Period	Annual $\frac{w}{r}$ Change	Labor Share	Factor Prices	Bias	Labor Share	Factor Prices	Bias
Real	1970-1999	1.97	-0.25	0.10	-0.35	-7.61	3.0	-10.6
Real	2000-2010	1.35	-0.79	0.09	-0.88	-8.73	0.94	-9.66
Nominal	1970-1999	1.13	-0.25	0.04	-0.30	-7.61	1.33	-8.94
Nominal	2000-2010	-0.14	-0.79	-0.02	-0.77	-8.73	-0.21	-8.52

Note: The factor price and bias contributions are as defined in the text. Annual Contributions are in percentage points per year and Cumulative Contributions are in percentage points.

F.4 Labor Share from Production Data

Our benchmark analysis decomposed labor's share of income as measured in the national accounts. This data is built from manufacturing firms. Alternatively, we could analyze the changes in labor share as measured from production data built from manufacturing plants. We will briefly describe the advantages of each and why the analysis based on national accounts is our preferred measure.

The national accounts is built from firm data, so it includes all establishments (including non-manufacturing establishments) of manufacturing firms. This data contains measures of overall labor compensation.

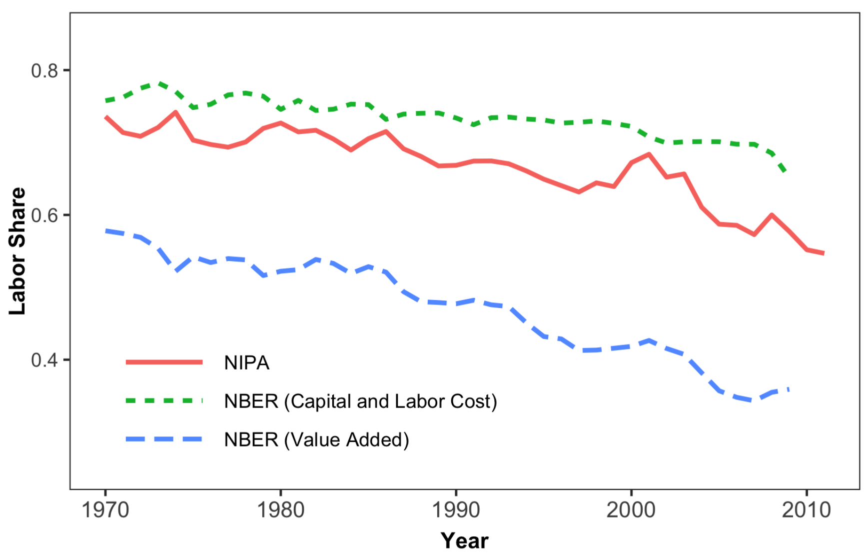
The production data from the NBER CES production database is built from the same manufacturing plant database that we used to compute the aggregate elasticity. Because the aggregate production data does not include benefits, in each year we adjust the payments to labor by the ratio of total compensation to wages and salaries for manufacturing from NIPA.

We prefer using the labor share from the national accounts for two reasons. First, it makes our study comparable to the rest of the literature that has studied the labor share. Second, the production data only includes expenses incurred at the plant level, such as energy and materials costs. It does not include expenses such as advertising, research and development not conducted at the plant, and all expenses at the corporate headquarters. The absence of these expense means that value added, and hence our residual "profit", are both overstated and may have different trends over time.

Nevertheless, we examine the change in the labor share and its components under two alternatives. First, we perform the same analysis as in the text, decomposing the change in the labor share of value added as measured in the production data. Second, we decompose the change in labor's share of the total expenditure on capital and labor, $d\frac{s_l}{s_l+s_k}$, into the contribution from factor prices and the contribution from biased technical change. We believe the latter is more comparable across the two sources.

Figure F.1 displays the NIPA time series from the main paper against the labor share from the NBER productivity database defined using both alternatives. The NBER measure of labor based on value added falls from 0.58 in 1970 to 0.36 in 2009, while the NBER measure of labor based on capital and labor cost declines from 0.76 in 1970 to 0.65 in 1999. The NIPA series declines from 0.74 in 1970 to 0.58 in 1999.

Figure F.1 Labor Share over Time: NIPA vs. NBER



Note: The red solid line is the labor share for manufacturing based on data from the BLS Multifactor Productivity series. The green short dashed line is the labor share of labor and capital costs for manufacturing using data from the NBER Productivity database. The blue long dashed line is the labor share of value added for manufacturing using data from the NBER Productivity database.

Table F.4 displays the change in the labor share and its components under two alternatives. In addition, we report the same statistics using the NIPA data.

Labor's share of value added in the production data declined at about the same rate between 1970-1999 and in the 2000s at 0.54 percentage points year from 1970-1999 and 0.57 percentage points per year from 2000-2019. About 75% of the change in the bias measured using labor's share of value added happens from 1970-1999.

In contrast, labor's share of capital/labor cost falls much faster in the 2000s, with an annual decline of 0.33 percentage points from 1970-1999 and 0.91 percentage points from 2000-2009. It is

Table F.4 Contributions to Labor Share Change using Production Data

Period	Annual Contribution Labor Share	Annual Contribution Factor Prices	Annual Contribution Bias	Cumulative Contribution Labor Share	Cumulative Contribution Factor Prices	Cumulative Contribution Bias
			Labor's Share of Value Added			
1970-1999	-0.54	0.11	-0.65	-16.29	3.23	-19.51
2000-2009	-0.57	0.10	-0.66	-5.67	0.95	-6.62
			Labor's Share of Capital and Labor Cost			
1970-1999	-0.17	0.16	-0.33	-5.11	4.78	-9.89
2000-2009	-0.75	0.16	-0.91	-7.52	1.57	-9.09
			Labor's Share of Value Added, NIPA			
1970-1999	-0.25	0.10	-0.35	-7.61	3.00	-10.62
2000-2009	-0.62	0.07	-0.69	-6.20	0.71	-6.91

Note: The factor price and bias contributions are as defined in the text. Annual Contributions are in percentage points per year and Cumulative Contributions are in percentage points.

thus more consistent both qualitatively and quantitatively with the overall pattern using national accounts.

G Additional Results for Section 3

This appendix describes some additional theoretical and quantitative results to complement Section 3. Web Appendix G.1 describes local elasticities of substitution and Web Appendix G.2.1 derives a preliminary result under the assumption that each plant's production function is homothetic. The assumption of constant returns to scale is relaxed in Web Appendix G.2.2. Web Appendix G.2.3 generalizes the demand system to allow for arbitrary elasticities of demand and imperfect pass-through. Web Appendix G.2.4 relaxes the assumption that production functions are homothetic.

For this section, we use the following notation for relative factor prices: $\omega \equiv \frac{w}{r}$ and $\mathfrak{q} \equiv \frac{q}{r}$. In addition, we define $p_{ni} \equiv P_{ni}/r$ and $p_n \equiv P_n/r$ to be plant i's and industry n's prices respectively normalized by the rental rate. It will also be useful to define i's cost function (normalized by r) to be

$$z_{ni}(Y_{ni}, \omega, \mathfrak{q}) = \min_{K_{ni}, L_{ni}, M_{ni}} K_{ni} + \omega L_{ni} + \mathfrak{q} M_{ni} \qquad \text{subject to} \qquad F_{ni}(K_{ni}, L_{ni}, M_{ni}) \geq Y_{ni}$$

As in Appendix A, two results will be used repeatedly. First, Shephard's lemma implies that for each i:

$$(1 - s_{ni}^M)(1 - \alpha_{ni}) = \frac{z_{ni\omega}(Y_{ni}, \omega, \mathfrak{q})\omega}{z_{ni}(Y_{ni}, \omega, \mathfrak{q})} \tag{G.1}$$

$$s_{ni}^M = \frac{z_{ni\mathfrak{q}}(Y_{ni}, \omega, \mathfrak{q})\mathfrak{q}}{z_{ni}(Y_{ni}, \omega, \mathfrak{q})} \tag{G.2}$$

Second, $\alpha_n = \sum_{i \in I_n} \alpha_{ni}\theta_{ni}$, so for any quantity κ_n,

$$\sum_{i \in I_n} (\alpha_{ni} - \alpha_n)\kappa_n \theta_{ni} = 0 \tag{G.3}$$

G.1 Locally-Defined Elasticities

In our baseline analysis we assumed that plant i produced using a nested CES production function of the form

$$F_{ni}(K_{ni}, L_{ni}, M_{ni}) = \left(\left[(A_{ni}K_{ni})^{\frac{\sigma-1}{\sigma}} + (B_{ni}L_{ni})^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}\frac{\zeta-1}{\zeta}} + (C_{ni}M_{ni})^{\frac{\zeta-1}{\zeta}} \right)^{\frac{\zeta}{\zeta-1}}$$

In that context, σ was i's elasticity of substitution between capital and labor and ζ was i's elasticity of substitution between materials and i's capital-labor bundle.

When i's production function does not take this parametric form, we define local elasticities of substitution. Suppose that i produces using the production function $Y_{ni} = F_{ni}(K_{ni}, L_{ni}, M_{ni})$ with corresponding cost function z_{ni}. We define σ_{ni} and ζ_{ni} to satisfy

$$\sigma_{ni} - 1 = \left. \frac{d \ln \frac{\alpha_{ni}}{1-\alpha_{ni}}}{d \ln \omega} \right|_{Y_{ni} \text{ is constant}}$$

$$(\alpha_{ni} - \alpha^M)(\zeta_{ni} - 1) = \left. \frac{d \ln \frac{1-s_{ni}^M}{s_{ni}^M}}{d \ln \omega} \right|_{Y_{ni} \text{ is constant}}$$

σ_{ni} and ζ_{ni} measure how i's relative factor usage changes in response to changes in relative factor prices holding i's output fixed (as one moves along an isoquant). That output remains fixed is relevant only if production functions are non-homothetic, in which case a change in a plant's scale would alter its relative factor usage. This section derives expressions for σ_{ni} and ζ_{ni} in terms of i's cost function.

Claim G.1 *σ_{ni} and ζ_{ni} satisfy*

$$\begin{aligned}(\alpha_{ni}-\alpha^M)\zeta_{ni} &= -\frac{1}{1-s_{ni}^M}\left[\frac{z_{ni\mathfrak{q}\omega}\omega}{z_{ni\mathfrak{q}}}+\frac{z_{ni\mathfrak{q}\mathfrak{q}}\mathfrak{q}}{z_{ni\mathfrak{q}}}(1-\alpha^M)\right]\\ \sigma_{ni} &= -\frac{1}{\alpha_{ni}}\left\{\frac{z_{ni\omega\omega}\omega}{z_{ni\omega}}+\frac{z_{ni\omega\mathfrak{q}}\mathfrak{q}}{z_{ni\omega}}(1-\alpha^M)+\frac{s_{ni}^M}{1-s_{ni}^M}\left[\frac{z_{ni\mathfrak{q}\omega}\omega}{z_{ni\mathfrak{q}}}+\frac{z_{ni\mathfrak{q}\mathfrak{q}}\mathfrak{q}}{z_{ni\mathfrak{q}}}(1-\alpha^M)\right]\right\}\end{aligned}$$

Proof. Differentiating (G.2) and (G.1) with respect to ω gives

$$\begin{aligned}\frac{d\ln s_{ni}^M}{d\ln\omega} &= \left\{\begin{array}{c}\frac{z_{ni\mathfrak{q}Y}Y_{ni}}{z_{ni\mathfrak{q}}}\frac{d\ln Y_{ni}}{d\ln\omega}+\frac{z_{ni\mathfrak{q}\omega}\omega}{z_{ni\mathfrak{q}}}+\frac{z_{ni\mathfrak{q}\mathfrak{q}}\mathfrak{q}}{z_{ni\mathfrak{q}}}\frac{d\ln\mathfrak{q}}{d\ln\omega}+\frac{d\ln\mathfrak{q}}{d\ln\omega}\\ -\frac{z_{niY}Y_{ni}}{z_{ni}}\frac{d\ln Y_{ni}}{d\ln\omega}-\frac{z_{ni\omega}\omega}{z_{ni}}-\frac{z_{ni\mathfrak{q}}\mathfrak{q}}{z_{ni}}\frac{d\ln\mathfrak{q}}{d\ln\omega}\end{array}\right\}\\ \frac{d\ln(1-s_{ni}^M)}{d\ln\omega}+\frac{d\ln(1-\alpha_{ni})}{d\ln\omega} &= \left\{\begin{array}{c}\frac{z_{ni\omega Y}Y_{ni}}{z_{ni\omega}}\frac{d\ln Y_{ni}}{d\ln\omega}+\frac{z_{ni\omega\omega}\omega}{z_{ni\omega}}+\frac{z_{ni\omega\mathfrak{q}}\mathfrak{q}}{z_{ni\omega}}\frac{d\ln\mathfrak{q}}{d\ln\omega}+1\\ -\frac{z_{niY}Y_{ni}}{z_{ni}}\frac{d\ln Y_{ni}}{d\ln\omega}-\frac{z_{ni\omega}\omega}{z_{ni}}-\frac{z_{ni\mathfrak{q}}\mathfrak{q}}{z_{ni}}\frac{d\ln\mathfrak{q}}{d\ln\omega}\end{array}\right\}\end{aligned}$$

Imposing $\frac{d\ln Y_{ni}}{d\ln\omega}=0$, $\frac{d\ln\mathfrak{q}}{d\ln\omega}=1-\alpha^M$, and Shephard's lemma, these equations can be written as

$$\begin{aligned}\frac{d\ln s_{ni}^M}{d\ln\omega} &= \frac{z_{ni\mathfrak{q}\omega}\omega}{z_{ni\mathfrak{q}}}+\frac{z_{ni\mathfrak{q}\mathfrak{q}}\mathfrak{q}}{z_{ni\mathfrak{q}}}(1-\alpha^M)+(1-\alpha^M)-(1-s_{ni}^M)(1-\alpha_{ni})-s_{ni}^M(1-\alpha^M)\\ \frac{d\ln(1-s_{ni}^M)}{d\ln\omega}+\frac{d\ln(1-\alpha_{ni})}{d\ln\omega} &= \frac{z_{ni\omega\omega}\omega}{z_{ni\omega}}+\frac{z_{ni\omega\mathfrak{q}}\mathfrak{q}}{z_{ni\omega}}(1-\alpha^M)+1-(1-s_{ni}^M)(1-\alpha_{ni})-s_{ni}^M(1-\alpha^M)\end{aligned}$$

Simplifying yields

$$\begin{aligned}\frac{d\ln s_{ni}^M}{d\ln\omega} &= \frac{z_{ni\mathfrak{q}\omega}\omega}{z_{ni\mathfrak{q}}}+\frac{z_{ni\mathfrak{q}\mathfrak{q}}\mathfrak{q}}{z_{ni\mathfrak{q}}}(1-\alpha^M)+(1-s_{ni}^M)(\alpha_{ni}-\alpha^M)\\ \frac{d\ln(1-s_{ni}^M)}{d\ln\omega}+\frac{d\ln(1-\alpha_{ni})}{d\ln\omega} &= \frac{z_{ni\omega\omega}\omega}{z_{ni\omega}}+\frac{z_{ni\omega\mathfrak{q}}\mathfrak{q}}{z_{ni\omega}}(1-\alpha^M)+\alpha_{ni}-s_{ni}^M(\alpha_{ni}-\alpha^M)\end{aligned}$$

Using $\frac{d\ln 1-s_{ni}^M}{d\ln\omega}=-\frac{s_{ni}^M}{1-s_{ni}^M}\frac{d\ln s_{ni}^M}{d\ln\omega}$ and plugging the first into the second yields

$$\frac{d\ln(1-\alpha_{ni})}{d\ln\omega} = \frac{z_{ni\omega\omega}\omega}{z_{ni\omega}}+\frac{z_{ni\omega\mathfrak{q}}\mathfrak{q}}{z_{ni\omega}}(1-\alpha^M)+\frac{s_{ni}^M}{1-s_{ni}^M}\left[\frac{z_{ni\mathfrak{q}\omega}\omega}{z_{ni\mathfrak{q}}}+\frac{z_{ni\mathfrak{q}\mathfrak{q}}\mathfrak{q}}{z_{ni\mathfrak{q}}}(1-\alpha^M)\right]+\alpha_{ni}$$

Finally, the definitions of the elasticities imply $\sigma_{ni}-1=-\frac{1}{\alpha_{ni}}\frac{d\ln(1-\alpha_{ni})}{d\ln\omega}$ and $(\alpha_{ni}-\alpha^M)(\zeta_{ni}-1)=-\frac{1}{1-s_{ni}^M}\frac{d\ln s_{ni}^M}{d\ln\omega}$, so that

$$\begin{aligned}(\alpha_{ni}-\alpha^M)\zeta_{ni} &= -\frac{1}{1-s_{ni}^M}\left[\frac{z_{ni\mathfrak{q}\omega}\omega}{z_{ni\mathfrak{q}}}+\frac{z_{ni\mathfrak{q}\mathfrak{q}}\mathfrak{q}}{z_{ni\mathfrak{q}}}(1-\alpha^M)\right]\\ \sigma_{ni} &= -\frac{1}{\alpha_{ni}}\left\{\frac{z_{ni\omega\omega}\omega}{z_{ni\omega}}+\frac{z_{ni\omega\mathfrak{q}}\mathfrak{q}}{z_{ni\omega}}(1-\alpha^M)+\frac{s_{ni}^M}{1-s_{ni}^M}\left[\frac{z_{ni\mathfrak{q}\omega}\omega}{z_{ni\mathfrak{q}}}+\frac{z_{ni\mathfrak{q}\mathfrak{q}}\mathfrak{q}}{z_{ni\mathfrak{q}}}(1-\alpha^M)\right]\right\}\end{aligned}$$

■

G.2 Elasticity of Demand and Returns to Scale

G.2.1 Industry Substitution and Within-Plant Substitution

We now define plant i's local returns to scale to be $\gamma_{ni} = \left[\frac{z_{niY}Y_{ni}}{z_{ni}}\right]^{-1}$. The next lemma characterizes the within-plant components of industry substitution.

Lemma G.1 *Suppose that plant i produces using the homothetic production function F_{ni}. The industry elasticity of substitution for industry n, σ_n^N, can be written as*

$$\sigma_n^N = (1-\chi_n)\bar{\sigma}_n + \chi_n \bar{s_n^M}\bar{\zeta}_n + \frac{\sum_{i\in I_n}(\alpha_{ni}-\alpha_n)\theta_{ni}\frac{1}{\gamma_{ni}}\frac{d\ln Y_{ni}}{d\ln\omega}}{\alpha_n(1-\alpha_n)}$$

where $\bar{\zeta}_n \equiv \sum_{i\in I_n}\frac{(\alpha_{ni}-\alpha_n)(\alpha_{ni}-\alpha^M)s_{ni}^M}{\sum_{j\in I_n}(\alpha_{nj}-\alpha_n)(\alpha_{nj}-\alpha^M)s_{nj}^M}\zeta_{ni}$ *and* $\bar{s}_n^M \equiv \sum_{i\in I_n}\frac{(\alpha_{ni}-\alpha_n)(\alpha_{ni}-\alpha^M)}{\sum_{j\in I_n}(\alpha_{nj}-\alpha_n)(\alpha_{nj}-\alpha^M)}s_{ni}^M$

Proof. Following the steps of the proof of Proposition 1′, we have

$$\begin{aligned}
\sigma_n^N &= (1-\chi_n)\sigma_n + \frac{\sum_{i\in I_n}(\alpha_{ni}-\alpha_n)\frac{d\theta_{ni}}{d\ln\omega}}{\alpha_n(1-\alpha_n)} + \chi_n & \text{(G.4)}\\
\theta_{ni} &= \frac{rK_{ni}+wL_{ni}}{\sum_{j\in I_n} rK_{nj}+wL_{nj}} = \frac{(1-s_{ni}^M)z_{ni}}{\sum_{j\in I_n}(1-s_{nj}^M)z_{nj}}\\
\frac{d\ln(1-s_{ni}^M)}{d\ln\omega} &= s_{ni}^M(\zeta_{ni}-1)(\alpha_{ni}-\alpha^M)
\end{aligned}$$

The change in i's expenditure on all inputs depends on its return to scale and its expenditure shares:

$$\begin{aligned}
\frac{d\ln z_{ni}(Y_{ni},\omega,\mathfrak{q})}{d\ln\omega} &= \frac{Y_{ni}z_{niY}}{z_{ni}}\frac{d\ln Y_{ni}}{d\ln\omega} + \frac{z_{ni\omega}\omega}{z_{ni}} + \frac{z_{ni\mathfrak{q}}\mathfrak{q}}{z_{ni}}\frac{d\ln\mathfrak{q}}{d\ln\omega}\\
&= \frac{1}{\gamma_{ni}}\frac{d\ln Y_{ni}}{d\ln\omega} + (1-s_{ni}^M)(1-\alpha_{ni}) + s_{ni}^M(1-\alpha^M)\\
&= \frac{1}{\gamma_{ni}}\frac{d\ln Y_{ni}}{d\ln\omega} + (1-\alpha_{ni}) + s_{ni}^M(\alpha_{ni}-\alpha^M) & \text{(G.5)}
\end{aligned}$$

Putting these pieces together, since $\sum_{i\in I_n}(\alpha_{ni}-\alpha_n)\theta_{ni}\frac{d\ln\sum_{j\in I_n}(1-s_{nj}^M)z_{nj}}{d\ln\omega} = 0$, we have

$$\begin{aligned}
\sum_{i\in I_n}(\alpha_{ni}-\alpha_n)\theta_{ni}\frac{d\ln\theta_{ni}}{d\ln\omega} &= \sum_{i\in I_n}(\alpha_{ni}-\alpha_n)\theta_{ni}\left[\frac{d\ln 1-s_{ni}^M}{d\ln\omega} + \frac{d\ln z_{ni}}{d\ln\omega}\right]\\
&= \sum_{i\in I_n}(\alpha_{ni}-\alpha_n)\theta_{ni}\left[\begin{array}{c} s_{ni}^M(\zeta_{ni}-1)(\alpha_{ni}-\alpha^M) + \frac{1}{\gamma_{ni}}\frac{d\ln Y_{ni}}{d\ln\omega}\\ +(1-\alpha_{ni}) + s_{ni}^M(\alpha_{ni}-\alpha^M)\end{array}\right]\\
&= \sum_{i\in I_n}(\alpha_{ni}-\alpha_n)\theta_{ni}\left[s_{ni}^M\zeta_{ni}(\alpha_{ni}-\alpha^M) + \frac{1}{\gamma_{ni}}\frac{d\ln Y_{ni}}{d\ln\omega} + (1-\alpha_{ni})\right]
\end{aligned}$$

Using the definitions of $\bar{\zeta}_n$ and $\bar{s}_n^M$, this becomes

$$\sum_{i\in I_n}(\alpha_{ni}-\alpha_n)\theta_{ni}\frac{d\ln\theta_{ni}}{d\ln\omega} = \sum_{i\in I_n}(\alpha_{ni}-\alpha_n)\theta_{ni}\left[\bar{s}_n^M\bar{\zeta}_n(\alpha_{ni}-\alpha^M)+\frac{1}{\gamma_{ni}}\frac{d\ln Y_{ni}}{d\ln\omega}+(1-\alpha_{ni})\right]$$

Using the fact that for any constant κ, $\sum_{i\in I_n}(\alpha_{ni}-\alpha_n)\theta_{ni}\kappa=0$, we can write this as

$$\sum_{i\in I_n}(\alpha_{ni}-\alpha_n)\theta_{ni}\left[\bar{s}_n^M\bar{\zeta}_n(\alpha_{ni}-\alpha_n)+\frac{1}{\gamma_{ni}}\frac{d\ln Y_{ni}}{d\ln\omega}-(\alpha_{ni}-\alpha_n)\right]$$

Finally, we can plug this back into (G.4) to get

$$\sigma_n^N=(1-\chi_n)\bar{\sigma}_n+\chi_n\bar{s}_n^M\bar{\zeta}_n+\frac{\sum_{i\in I_n}(\alpha_{ni}-\alpha_n)\theta_{ni}\frac{1}{\gamma_{ni}}\frac{d\ln Y_{ni}}{d\ln\omega}}{\alpha_n(1-\alpha_n)}$$

■

G.2.2 Returns to Scale

This section relaxes the assumption that each plant's production function exhibits constant returns to scale.

Claim G.2 *Suppose that plant i produces using the production function $Y_{ni}=F_{ni}(K_{ni},L_{ni},M_{ni})=G_{ni}(K_{ni},L_{ni},M_{ni})^{\gamma}$, where G_{ni} has constant returns to scale and $\gamma\le\frac{\varepsilon_n}{\varepsilon_n-1}$. Let $x=\frac{\varepsilon_n}{\varepsilon_n+\gamma(1-\varepsilon_n)}$. Then the industry elasticity of substitution is*

$$\sigma_n=(1-\chi_n)\bar{\sigma}_n+\chi_n\left[\bar{s}_n^M\bar{\zeta}_n+(1-\bar{s}_n^M)x\right]$$

and the revenue-cost ratio is $\frac{P_{ni}Y_{ni}}{rK_{ni}+wL_{ni}+qM_{ni}}=\frac{x}{x-1}$.

Proof. Plant i's optimal price is $p_{ni}=\frac{\varepsilon_n}{\varepsilon_n-1}z_{niY}(Y_{ni},\omega,\mathfrak{q})$, so differentiating yields

$$\frac{d\ln p_{ni}}{d\ln\omega}=\frac{z_{niYY}Y_{ni}}{z_{niY}}\frac{d\ln Y_{ni}}{d\ln\omega}+\frac{z_{niY\omega}\omega}{z_{niY}}+\frac{z_{niY\mathfrak{q}}\mathfrak{q}}{z_{niY}}\frac{d\ln\mathfrak{q}}{d\ln\omega}$$

The production function implies that $\frac{z_{niYY}Y_{ni}}{z_{niY}}=\frac{1}{\gamma}-1$, $\frac{z_{niY\omega}\omega}{z_{niY}}=(1-\alpha_{ni})(1-s_{ni}^M)$, and $\frac{z_{niY\mathfrak{q}}\mathfrak{q}}{z_{niY}}=s_{ni}^M$, so this can be written as

$$\frac{d\ln p_{ni}}{d\ln\omega}=\left(\frac{1}{\gamma}-1\right)\frac{d\ln Y_{ni}}{d\ln\omega}+(1-\alpha_{ni})(1-s_{ni}^M)+s_{ni}^M(1-\alpha^M)$$

The change in i's output is then

$$\begin{aligned}\frac{d\ln Y_{ni}}{d\ln\omega} &= -\varepsilon_n\frac{d\ln p_{ni}}{d\ln\omega}+\frac{d\ln Y_n p_n^{\varepsilon_n}}{d\ln\omega}\\ &= -\varepsilon_n\left(\frac{1}{\gamma}-1\right)\frac{d\ln Y_{ni}}{d\ln\omega}-\varepsilon_n\left[(1-\alpha_{ni})(1-s_{ni}^M)+s_{ni}^M(1-\alpha^M)\right]+\left[\frac{d\ln Y_n p_n^{\varepsilon_n}}{d\ln\omega}\right]\end{aligned}$$

This can be rearranged as

$$\frac{d\ln Y_{ni}}{d\ln\omega}=\gamma x\left[(\alpha_{ni}-\alpha_n)-s_{ni}^M(\alpha_{ni}-\alpha^M)\right]+\frac{x\gamma}{\varepsilon_n}\left[\frac{d\ln Y_n p_n^{\varepsilon_n}}{d\ln\omega}-\varepsilon_n(1-\alpha_n)\right]$$

Using Lemma G.1 and the fact that $\sum_{i\in I_n}(\alpha_{ni}-\alpha_n)\theta_{ni}\left[\frac{d\ln Y_n p_n^{\varepsilon_n}}{d\ln\omega}-\varepsilon_n(1-\alpha_n)\right]=0$ gives

$$\begin{aligned}\sigma_n^N &= (1-\chi_n)\bar{\sigma}_n+\chi_n\bar{s}_n^M\bar{\zeta}_n+\frac{\sum_{i\in I_n}(\alpha_{ni}-\alpha_n)\theta_{ni}x\left[(\alpha_{ni}-\alpha_n)-s_{ni}^M(\alpha_{ni}-\alpha^M)\right]}{\alpha_n(1-\alpha_n)}\\ &= (1-\chi_n)\bar{\sigma}_n+\chi_n\bar{s}_n^M\bar{\zeta}_n+\frac{\sum_{i\in I_n}(\alpha_{ni}-\alpha_n)\theta_{ni}x\left[(\alpha_{ni}-\alpha_n)-\bar{s}_n^M(\alpha_{ni}-\alpha^M)\right]}{\alpha_n(1-\alpha_n)}\end{aligned}$$

where the second line uses the definition of $\bar{s}_n^M$. The desired result follows using $\sum_{i\in I_n}(\alpha_{ni}-\alpha_n)\theta_{ni}\bar{s}_n^M(\alpha^M-\alpha_n)=0$ and the definition of χ_n.

Finally, since $p_{ni}=\frac{\varepsilon_n}{\varepsilon_n-1}z_{niY}$, the revenue cost ratio is

$$\frac{P_{ni}Y_{ni}}{rK_{ni}+wL_{ni}+qM_{ni}}=\frac{p_{ni}Y_{ni}}{z_{ni}}=\frac{\varepsilon_n}{\varepsilon_n-1}\frac{z_{niY}Y_{ni}}{z_{ni}}=\frac{\varepsilon_n}{\varepsilon_n-1}\frac{1}{\gamma}=\frac{x}{x-1}$$

■

G.2.3 Demand

In this section we generalize the demand system to a class of homothetic demand systems in which demand for each good is strongly separable. While this class nests Dixit-Stiglitz demand, it allows for arbitrary demand elasticities and pass through rates. An industry aggregate Y_n is defined to satisfy

$$1=\sum_{i\in I_n}H_{ni}\left(Y_{ni}/Y_n\right) \tag{G.6}$$

where each H_{ni} is positive, smooth, increasing, and concave. If P_n is the ideal price index associated with Y_n, then cost minimization implies $\frac{P_{ni}}{P_n}=H'_{ni}\left(\frac{Y_{ni}}{Y_n}\right)$. Define the inverse of H'_{ni} to be $h_{ni}(\cdot)=H'^{-1}_{ni}(\cdot)$. i faces a demand curve; to find its elasticity of demand, we can differentiate:

$$d\ln Y_{ni}/Y_n=-\varepsilon_{ni}(P_{ni}/P_n)d\ln P_{ni}/P_n \tag{G.7}$$

where the elasticity of demand is $\varepsilon_{ni}(x)\equiv-\frac{h'_{ni}(x)x}{h_{ni}(x)}$. The optimal markup chosen by i will satisfy $\mu_{ni}(P_{ni}/P_n)=\frac{\varepsilon_{ni}(P_{ni}/P_n)}{\varepsilon_{ni}(P_{ni}/P_n)-1}$. It will be useful to define b_{ni} to be i's local relative rate of pass through: the responsiveness of P_{ni} to a change in i's marginal cost. Since $P_{ni}=\mu(P_{ni}/P_n)\times\text{mc}_{ni}$, then $\frac{d\ln P_{ni}}{d\ln\text{mc}_{ni}}=\frac{P_{ni}/P_n\mu'_{ni}}{\mu_{ni}}\frac{d\ln P_{ni}}{d\ln\text{mc}_{ni}}+1$, so that $b_{ni}(x)\equiv\frac{1}{1-\frac{x\mu'_{ni}(x)}{\mu_{ni}(x)}}$.

Lastly, we define $\alpha_n^P\equiv1-\frac{d\ln p_n}{d\ln\omega}$ to be the response of the ideal price index to a change in relative factor prices. The following claim describes the industry elasticity of substitution.

Claim G.3 *Suppose that each F_{ni} exhibits constant returns to scale and the demand structure in industry n satisfies (G.6). Then the industry elasticity is*

$$\sigma_n^N = (1-\chi_n)\bar{\sigma}_n+\chi_n\bar{s}_n^M\bar{\zeta}_n+\chi_n(1-\bar{s}_n^M)\bar{x}_n$$

where

$$\bar{x}_n \equiv \frac{\sum_{i\in I_n}(\alpha_{ni}-\alpha_n)\theta_{ni}\left[(\alpha_{ni}-\alpha_n^P)-s_{ni}^M(\alpha_{ni}-\alpha^M)\right]\varepsilon_{ni}b_{ni}}{\sum_{i\in I_n}(\alpha_{ni}-\alpha_n)\theta_{ni}\left[(\alpha_{ni}-\alpha_n^P)-s_{ni}^M(\alpha_{ni}-\alpha^M)\right]}$$

$$\alpha_n^P = \frac{\sum_{i\in I_n}P_{ni}Y_{ni}\varepsilon_{ni}b_{ni}\left[\alpha_{ni}-s_{ni}^M(\alpha_{ni}-\alpha^M)\right]}{\sum_{i\in I_n}P_{ni}Y_{ni}\varepsilon_{ni}b_{ni}}$$

Proof. Optimal price setting implies $p_{ni}=\mu_i(p_{ni}/p_n)z_{niY}$. Taking logs and differentiating gives

$$\frac{d\ln p_{ni}/p_n}{d\ln\omega}=\frac{\mu_i'(p_{ni}/p_n)p_{ni}/p_n}{\mu_i(p_{ni}/p_n)}\frac{d\ln p_{ni}/p_n}{d\ln\omega}+\frac{d\ln z_{niY}}{d\ln\omega}-\frac{d\ln p_n}{d\ln\omega}$$

Constant returns to scale implies $\frac{d\ln z_{niY}}{d\ln\omega}=(1-\alpha_{ni})(1-s_{ni}^M)+s_{ni}^M(1-\alpha^M)=(1-\alpha_{ni})+s_{ni}^M(\alpha_{ni}-\alpha^M)$, so this can be written as

$$\frac{d\ln p_{ni}/p_n}{d\ln\omega}=b_{ni}\left[s_{ni}^M(\alpha_{ni}-\alpha^M)-(\alpha_{ni}-\alpha_n^P)\right] \tag{G.8}$$

The change in output is then

$$\frac{d\ln Y_{ni}/Y_n}{d\ln\omega} = -\varepsilon_{ni}\frac{d\ln p_{ni}/p_n}{d\ln\omega}=\varepsilon_{ni}b_{ni}\left[(\alpha_{ni}-\alpha_n^P)-s_{ni}^M(\alpha_{ni}-\alpha^M)\right]$$

To get at the aggregate elasticity, we compute the following

$$\begin{aligned}
\sum_{i\in I_n}(\alpha_{ni}-\alpha_n)\theta_{ni}\frac{d\ln Y_{ni}}{d\ln\omega} &= \sum_{i\in I_n}(\alpha_{ni}-\alpha_n)\theta_{ni}\frac{d\ln Y_{ni}/Y_n}{d\ln\omega}\\
&= \sum_{i\in I_n}(\alpha_{ni}-\alpha_n)\theta_{ni}\varepsilon_{ni}b_{ni}\left[(\alpha_{ni}-\alpha_n^P)-s_{ni}^M(\alpha_{ni}-\alpha^M)\right]\\
&= \sum_{i\in I_n}(\alpha_{ni}-\alpha_n)\theta_{ni}\bar{x}_n\left[(\alpha_{ni}-\alpha_n^P)-s_{ni}^M(\alpha_{ni}-\alpha^M)\right]\\
&= \sum_{i\in I_n}(\alpha_{ni}-\alpha_n)\theta_{ni}\bar{x}_n\left[(\alpha_{ni}-\alpha_n^P)-\bar{s}_n^M(\alpha_{ni}-\alpha^M)\right]\\
&= \sum_{i\in I_n}(\alpha_{ni}-\alpha_n)\theta_{ni}\bar{x}_n\left[(\alpha_{ni}-\alpha_n)-\bar{s}_n^M(\alpha_{ni}-\alpha_n)\right]\\
&= \alpha_n(1-\alpha_n)\chi_n(1-\bar{s}_n^M)\bar{x}_n
\end{aligned}$$

where the third equality uses the definition of $\bar{x}_n$ and the fourth uses the definition $\bar{s}_n^M$. This expression and Lemma G.1 give the desired result.

It remains only to compute α_n^P. Since $\sum_{i\in I_n}\frac{P_{ni}Y_{ni}}{P_nY_n}\frac{d\ln Y_{ni}/Y_n}{d\ln\omega}=0$, we can use (G.7) and (G.8) to write

$$0=\sum_{i\in I_n}\frac{P_{ni}Y_{ni}}{P_nY_n}\varepsilon_{ni}b_{ni}\left[s_{ni}^M(\alpha_{ni}-\alpha^M)-(\alpha_{ni}-\alpha_n^P)\right]$$

which simplifies to

$$\alpha_n^P=\frac{\sum_{i\in I_n}P_{ni}Y_{ni}\varepsilon_{ni}b_{ni}\left[\alpha_{ni}-s_{ni}^M(\alpha_{ni}-\alpha^M)\right]}{\sum_{i\in I_n}P_{ni}Y_{ni}\varepsilon_{ni}b_{ni}}$$

■

G.2.4 Non-Homothetic Production

This section analyzes how the industry elasticity of substitution is altered if production is non-homothetic. This requires a more careful definition of the elasticities of substitution. A change in factor prices will have a direct effect on a plant's choice of capital-labor ratio, and may have an indirect impact if the change in factor prices alters a plant's scale. We pursue an approach similar to Joan Robinson: we define a plant's elasticity of substitution to be how a change in relative factor prices alters the plant's capital-labor ratio holding output fixed. Similarly, an industry's elasticity of substitution is the response of the industry's capital labor ratio to a change in relative factor prices holding fixed the industry aggregate, Y_n.

We first characterize the plant-level elasticity of substitution, and then derive an expression for the industry level elasticity. In the interest of space, we restrict attention to the case in which plants do not use materials.

Just as $1-\alpha_{ni}\left(=\frac{z_{ni\omega}\omega}{z_{ni}}\right)$ is the labor share of i's cost, we define $\tilde{\alpha}_{ni}$ so that $1-\tilde{\alpha}_{ni}=\frac{z_{niY\omega}\omega}{z_{niY}}$, the labor share of i's marginal cost.

Since $1-\alpha_{ni}=\frac{z_{ni\omega}(Y_{ni},\omega)\omega}{z_{ni}(Y_{ni},\omega)}$, we have

$$d\ln(1-\alpha_{ni}) \;=\; \frac{z_{ni\omega Y}Y_{ni}}{z_{ni\omega}}d\ln Y_{ni}+\frac{z_{ni\omega\omega}\omega}{z_{ni\omega}}d\ln\omega+d\ln\omega-\frac{z_{niY}Y_{ni}}{z_{ni}}d\ln Y_{ni}-\frac{z_{ni\omega}\omega}{z_{ni}}$$

Since $\frac{z_{ni\omega Y}Y_{ni}}{z_{ni\omega}}=\frac{z_{ni\omega Y}\omega}{z_{niY}}\frac{z_{niY}Y_{ni}}{z_{ni}}$, this can be arranged as

$$d\ln(1-\alpha_{ni}) \;=\; \left(\frac{z_{ni\omega\omega}\omega}{z_{ni\omega}}+1-(1-\alpha_{ni})\right)d\ln\omega+\left(\frac{1-\tilde{\alpha}_{ni}}{1-\alpha_{ni}}-1\right)\frac{1}{\gamma_{ni}}d\ln Y_{ni}$$

Using $d\ln\frac{\alpha_{ni}}{1-\alpha_{ni}}=-\frac{1}{\alpha_{ni}}d\ln(1-\alpha_{ni})$, we have

$$d\ln\frac{\alpha_{ni}}{1-\alpha_{ni}}=\left(-\frac{1}{\alpha_{ni}}\frac{z_{ni\omega\omega}\omega}{z_{ni\omega}}-1\right)d\ln\omega+\frac{\tilde{\alpha}_{ni}-\alpha_{ni}}{\alpha_{ni}(1-\alpha_{ni})}\frac{1}{\gamma_{ni}}d\ln Y_{ni}$$

By definition, $\sigma_{ni}-1$ is the change in $\frac{\alpha_{ni}}{1-\alpha_{ni}}$ holding Y_{ni} fixed. The plant level elasticity of substitution is

$$\sigma_{ni}=-\frac{1}{\alpha_{ni}}\frac{z_{ni\omega\omega}\omega}{z_{ni\omega}}$$

and

$$d\ln\frac{\alpha_{ni}}{1-\alpha_{ni}}=(\sigma_{ni}-1)d\ln\omega+\frac{\tilde{\alpha}_{ni}-\alpha_{ni}}{\alpha_{ni}(1-\alpha_{ni})}\frac{1}{\gamma_{ni}}d\ln Y_{ni} \tag{G.9}$$

Claim G.4 *The industry elasticity is*

$$\sigma_n^N=(1-\chi_n)\bar{\sigma}_n+\tilde{\chi}_n\bar{x}_n$$

where χ_n *and* $\bar{\sigma}_n$ *are defined as in Lemma G.1 and*

$$\tilde{\chi}_n \;\equiv\; \sum_{i\in I_n}\frac{(\tilde{\alpha}_{ni}-\alpha_n)(\tilde{\alpha}_{ni}-\alpha_n^P)\theta_{ni}}{\alpha_n(1-\alpha_n)}$$

$$\bar{x}_n \;\equiv\; \frac{\sum_{i\in I_n}(\tilde{\alpha}_{ni}-\alpha_n)(\tilde{\alpha}_{ni}-\alpha_n^P)\theta_{ni}\frac{\varepsilon_n/\gamma_{ni}}{1+\varepsilon_n\frac{z_{niYY}Y_{ni}}{z_{niY}}}}{\sum_{i\in I_n}(\tilde{\alpha}_{ni}-\alpha_n)(\tilde{\alpha}_{ni}-\alpha_n^P)\theta_{ni}}$$

and α_n^P *is defined to satisfy* $1-\alpha_n^P = \frac{d\ln p_n}{d\ln\omega}$.

Proof. Following the same logic as in the benchmark, we have

$$d\ln\frac{\alpha_n}{1-\alpha_n} = \sum_{i\in I_n}\frac{\alpha_{ni}(1-\alpha_{ni})\theta_{ni}}{\alpha_n(1-\alpha_n)}d\ln\frac{\alpha_{ni}}{1-\alpha_{ni}} + \sum_{i\in I_n}\frac{(\alpha_{ni}-\alpha_n)}{\alpha_n(1-\alpha_n)}\theta_{ni}d\ln\theta_{ni}$$

Using (G.9), this becomes

$$\begin{aligned} d\ln\frac{\alpha_n}{1-\alpha_n} &= \sum_{i\in I_n}\frac{\alpha_{ni}(1-\alpha_{ni})\theta_{ni}}{\alpha_n(1-\alpha_n)}(\sigma_{ni}-1)d\ln\omega + \sum_{i\in I_n}\frac{(\tilde{\alpha}_{ni}-\alpha_{ni})\theta_{ni}}{\alpha_n(1-\alpha_n)}\frac{1}{\gamma_{ni}}d\ln Y_{ni} + \sum_{i\in I_n}\frac{(\alpha_{ni}-\alpha_n)}{\alpha_n(1-\alpha_n)}\theta_{ni}d\ln\theta_{ni} \\ &= (1-\chi_n)(\bar{\sigma}_n-1)d\ln\omega + \sum_{i\in I_n}\frac{(\tilde{\alpha}_{ni}-\alpha_{ni})\theta_{ni}}{\alpha_n(1-\alpha_n)}\frac{1}{\gamma_{ni}}d\ln Y_{ni} + \sum_{i\in I_n}\frac{(\alpha_{ni}-\alpha_n)}{\alpha_n(1-\alpha_n)}\theta_{ni}d\ln\theta_{ni} \end{aligned}$$

where the second line used the definitions of $\bar{\sigma}_n$ and χ_n. Since $\theta_{ni} = z_{ni}/\sum_{j\in I_n} z_{nj}$, we have

$$\begin{aligned} \sum_{i\in I_n}(\alpha_{ni}-\alpha_n)\theta_{ni}d\ln\theta_{ni} &= \sum_{i\in I_n}(\alpha_{ni}-\alpha_n)\theta_{ni}\left[\frac{z_{niY}Y_{ni}}{z_{ni}}d\ln Y + \frac{z_{ni\omega}\omega}{z_{ni}}d\ln\omega - d\ln\sum_{j\in I_n} z_{nj}\right] \\ &= \sum_{i\in I_n}(\alpha_{ni}-\alpha_n)\theta_{ni}\left[\frac{1}{\gamma_{ni}}d\ln Y + (1-\alpha_{ni})d\ln\omega\right] \end{aligned}$$

Plugging this in and combining coefficients gives

$$d\ln\frac{\alpha_n}{1-\alpha_n} = (1-\chi_n)(\bar{\sigma}_n-1)d\ln\omega + \sum_{i\in I_n}\frac{(\tilde{\alpha}_{ni}-\alpha_n)\theta_{ni}}{\alpha_n(1-\alpha_n)}\frac{1}{\gamma_{ni}}d\ln Y_{ni} + \sum_{i\in I_n}\frac{(\alpha_{ni}-\alpha_n)}{\alpha_n(1-\alpha_n)}\theta_{ni}(1-\alpha_{ni})d\ln\omega$$

One can easily verify that $\sum_{i\in I_n}\frac{(\alpha_{ni}-\alpha_n)(\alpha_{ni}-1)}{\alpha_n(1-\alpha_n)}\theta_{ni} = \chi_n$. This and $d\ln\frac{\alpha_n}{1-\alpha_n} = d\ln K_n/L_n - d\ln\omega$ imply

$$d\ln K_n/L_n = (1-\chi_n)\bar{\sigma}_n d\ln\omega + \sum_{i\in I_n}\frac{(\tilde{\alpha}_{ni}-\alpha_n)\theta_{ni}}{\alpha_n(1-\alpha_n)}\frac{1}{\gamma_{ni}}d\ln Y_{ni} \tag{G.10}$$

Finally we need to address the changes in scale. i's price is $p_{ni} = \frac{\varepsilon_n}{\varepsilon_n - 1}z_{niY}$, so the change in i's price is

$$\begin{aligned} d\ln p_{ni} &= \frac{z_{niYY}Y_{ni}}{z_{niY}}d\ln Y_{ni} + \frac{z_{niY\omega}\omega}{z_{niY}}d\ln\omega \\ &= \frac{z_{niYY}Y_{ni}}{z_{niY}}d\ln Y_{ni} + (1-\tilde{\alpha}_{ni})d\ln\omega \end{aligned}$$

If the change in the industry price index satisfies $d\ln p_n = (1-\alpha_n^P)d\ln\omega$, then the change in output is

$$
\begin{aligned}
d \ln Y_{ni} &= -\varepsilon_n d \ln \frac{p_{ni}}{p_n} + d \ln Y_n \\
&= -\varepsilon_n \left(\frac{z_{niYY} Y_{ni}}{z_{niY}} d \ln Y_{ni} + (1 - \tilde{\alpha}_{ni}) d \ln \omega - (1 - \alpha_n^P) d \ln \omega \right) + d \ln Y_n \\
&= \frac{\varepsilon_n (\tilde{\alpha}_{ni} - \alpha_n^P) d \ln \omega + d \ln Y_n}{1 + \varepsilon_n \frac{z_{niYY} Y_{ni}}{z_{niY}}}
\end{aligned}
$$

Using the definition of $\bar{x}_n$ and $\tilde{\chi}_n$, we therefore have that

$$
\begin{aligned}
d \ln K_n / L_n &= (1 - \chi_n) \bar{\sigma}_n d \ln \omega + \sum_{i \in I_n} \frac{(\tilde{\alpha}_{ni} - \alpha_n) \theta_{ni}}{\alpha_n (1 - \alpha_n)} \frac{1}{\gamma_{ni}} \frac{\varepsilon_n (\tilde{\alpha}_{ni} - \alpha_n^P) d \ln \omega + d \ln Y_n}{1 + \varepsilon_n \frac{z_{niYY} Y_{ni}}{z_{niY}}} \\
&= (1 - \chi_n) \bar{\sigma}_n d \ln \omega + \tilde{\chi}_n \bar{x}_n d \ln \omega + \sum_{i \in I_n} \frac{(\tilde{\alpha}_{ni} - \alpha_n) \theta_{ni}}{\alpha_n (1 - \alpha_n)} \frac{1/\gamma_{ni}}{1 + \varepsilon_n \frac{z_{niYY} Y_{ni}}{z_{niY}}} d \ln Y_n
\end{aligned}
$$

Since σ_n^N is defined to be the change in K_n/L_n in response to a change in ω holding fixed Y_n, we have

$$
\sigma_n^N = (1 - \chi_n) \bar{\sigma}_n + \tilde{\chi}_n \bar{x}_n
$$

■

H Entry and Exit

This section studies an economy with entry and exit by introducing entry and overhead costs. In doing this, we must address several issues. First, we have to specify which expenditures are measured in our data. For example, entry costs incurred before production are likely not measured in our data. Second, we have to specify the factor content of entry and overhead costs. In this section, we study several variations of these assumptions. For each, we derive an upper bound for the aggregate elasticity of substitution equal to or slightly above our baseline estimate, as well as a lower bound using our dynamic panel estimates.

H.1 Environment and Summary of Results

Entry and Overhead Costs Use Final Output

Consider an economy with a continuum of entrepreneurs. Each entrepreneur can draw a random technology τ from an exogenous distribution with CDF $T(\tau)$ by paying an entry cost of f^E units of final output. After observing the draw, she can operate a plant with the production function $F_\tau(K, L, M)$ if she is willing to pay an overhead cost of f^O units of final output. Each production function F_τ is assumed to exhibit constant returns to scale. We assume here that the overhead cost is measured as an expenditure on intermediate inputs in our data, but the entry cost is not.

For a plant with technology τ, let c^v_τ be the unit cost associated with the production function F_τ. Entrepreneurs follow a cutoff rule and operate the plant if variable profit outweighs the fixed operating cost. Free entry implies that cost of a productivity draw equals the expected profit, $Pf^E = \int \max\left\{(p_\tau - c^v_\tau)y_\tau - Pf^O, 0\right\} dT(\tau)$, where p_τ and y_τ are the optimal choices of price and quantity.

Let E_τ be an indicator of whether plant τ chooses to operate. Should the plant enter, we denote its capital share by α_τ and its expenditure on capital and labor as a fraction of the average expenditure by $\theta_\tau = \frac{rK_\tau + wL_\tau}{\int [rK_{\tilde{\tau}} + wL_{\tilde{\tau}}]E_{\tilde{\tau}} \mathrm{d}T(\tilde{\tau})}$. Thus the aggregate capital share can be expressed as $\alpha = \int \alpha_\tau \theta_\tau E_\tau dT(\tau)$. We show in Web Appendix H.2 that the aggregate capital labor elasticity is given by

$$\begin{aligned}\sigma^{agg} - 1 = &\frac{1}{\alpha(1-\alpha)} \int \frac{\mathrm{d}\alpha_\tau}{\mathrm{d}\ln w/r} \theta_\tau E_\tau \mathrm{d}T(\tau) + \frac{1}{\alpha(1-\alpha)} \int \frac{\mathrm{d}E_\tau}{\mathrm{d}\ln w/r} (\alpha_\tau - \alpha)\theta_\tau dT(\tau) \\ &+ \chi \bar{s}^M(\bar{\zeta} - 1) + \chi(1 - \bar{s}^M)(\varepsilon - 1) \qquad \text{(H.1)}\end{aligned}$$

where $\chi \equiv \frac{\int (\alpha_\tau - \alpha)^2 \theta_\tau E_\tau \mathrm{d}T(\tau)}{\alpha(1-\alpha)}$, s^M_τ is the share of observed expenditures (including both the operating cost and variable costs) spent on intermediate materials and ζ_τ captures substitution between intermediate and primary inputs, defined to satisfy $(\alpha - \alpha_\tau)(\zeta_\tau - 1)\frac{\mathrm{d}\ln \frac{s^M_\tau}{1-s^M_\tau}}{\mathrm{d}\ln w/r}$, $\bar{s}^M \equiv \frac{\int (\alpha_\tau - \alpha)(\alpha_\tau - \alpha^M)\theta_\tau s^M_\tau \mathrm{d}T(\tau)}{\int (\alpha_\tau - \alpha)(\alpha_\tau - \alpha^M)\theta_\tau \mathrm{d}T(\tau)}$, $\bar{\zeta} \equiv \frac{\int (\alpha_\tau - \alpha)(\alpha_\tau - \alpha^M)\theta_\tau s^M_\tau \zeta_\tau \mathrm{d}T(\tau)}{\int (\alpha_\tau - \alpha)(\alpha_\tau - \alpha^M)\theta_\tau s^M_\tau \mathrm{d}T(\tau)}$, and $\alpha^M \equiv \frac{\mathrm{d}\ln P/r}{\mathrm{d}\ln w/r}$.

The first term captures within-plant substitution between capital and labor. The second term captures the change in the aggregate capital share due to entry and exit; an increase in the wage induces labor-intensive plants to exit and capital-intensive plants to enter. The third term captures substitution between primary and intermediate inputs. The final term captures changes in plants' scales; an increase in the wage causes capital-intensive plants to expand relative to labor-intensive plants.

With this formula in hand, we now show that our baseline estimate of the aggregate elasticity of substitution is larger than the true aggregate elasticity. First, our estimated micro elasticity of substitution—-in particular the estimate of $\bar{\sigma}$ derived using α_τ as the dependent variable in (C.1)—incorporates the first two terms of (H.1), capturing both within-plant substitution and changes due to entry and exit. At root, our cross-sectional estimates capture how the average capital share varies with the local wage, and changes in this average reflect both the intensive and extensive margins. In Web Appendix H.3 we provide more detail, discuss how selection causes an upward bias similar to the weighted regressions in column 5 of Table C.2, and confirm these findings using Monte Carlo simulations in Web Appendix H.5.

Second the estimated micro elasticity of substitution between intermediate and primary inputs, $\hat{\zeta}$, reported in Table III, is larger than $\bar{\zeta}$. $\bar{\zeta}$ captures only the intensive margin—substitution within plants—while $\hat{\zeta}$ uses cross-sectional variation and incorporates both the intensive and extensive margins.

Finally, our baseline strategy overstates how a plant's scale responds to a change in its marginal cost because part of this cost—the overhead cost—is fixed. Formally, we had estimated this response from plants' ratios of revenue to cost (in our baseline model, this was a function of the elasticity of demand, $\frac{\varepsilon}{\varepsilon-1}$). Here, cost includes both variable and overhead components: $\frac{\hat{\varepsilon}_\tau}{\hat{\varepsilon}_\tau-1} = \frac{\frac{\varepsilon}{\varepsilon-1} c^v_\tau y_\tau}{c^v_\tau y_\tau + c^O f^O} < \frac{\varepsilon}{\varepsilon-1}$, or $\varepsilon < \hat{\varepsilon}_\tau$.

Together, these imply that our baseline estimate is an upper bound for the true aggregate elasticity. Using the restrictions that $\hat{\varepsilon} > 1$ and $\bar{\zeta} > 0$, the intensive margin effect (the first term in (H.1)) provides a conservative lower bound on the aggregate elasticity. We use our dynamic panel estimates to compute this lower bound. The implied range averages $[0.35, 0.65]$ across years.[13]

We also explore alternative assumptions about the factor content of overhead costs. If overhead costs used a plant's own output, then the upper bound for the aggregate elasticity is the same and the lower bound is slightly lower, and can be found by setting $\varepsilon = 0$. This averages 0.30 across years. If overhead costs used labor, the aggregate elasticity would include an extra term which captures the contribution of changes in the composition of expenditures between variable and overhead costs. However, this term is negative and quantitatively negligible, so the upper bound is the same as when the overhead cost uses final output, and the lower bound is slightly lower.

Foregone Labor

We now instead assume that both entry costs and overhead costs require the entrepreneur's labor, but these costs—the opportunity cost of the entrepreneur's time—do not appear on the plant's wage bill. In such a world, the measured capital share $\hat{\alpha}$, based on measured expenditures on labor and capital, differs from α, the true capital share incorporating unmeasured labor. Entry and overhead costs in unmeasured labor mean that the measured capital share is above the true capital share, so $\hat{\alpha} > \alpha$.

We then define two aggregate elasticities: $\hat{\sigma}^{agg} - 1 \equiv \frac{\mathrm{d}\ln \frac{\hat{\alpha}}{1-\hat{\alpha}}}{\mathrm{d}\ln w/r}$ captures changes in measured factor usage, while $\sigma^{agg} - 1 = \frac{\mathrm{d}\ln \frac{\alpha}{1-\alpha}}{\mathrm{d}\ln w/r}$ captures changes in true factor usage. $\hat{\sigma}^{agg}$ is relevant for questions about changes in national accounts, whereas σ^{agg} is relevant for questions such as the welfare cost of capital taxation. In practice, we show that the two elasticities are fairly close. The measured share elasticity $\hat{\sigma}^{agg}$ corresponds to our baseline estimate, while the resource-based

[13] The lower bound uses an intensive margin micro elasticity of substitution of 0.34 from column (2) of Table C.6. To derive the upper bound, we compute the aggregate elasticity in each year using our baseline formula but using the estimated cross-sectional elasticity from column (4) of Table C.2.

elasticity σ^{agg} is slightly higher than our baseline estimate.

To see the connection between the two, we define the following objects: $V \equiv \int c^v_\tau y_\tau E_\tau \mathrm{d}T(\tau)$ and $O \equiv \int wf^O E_\tau \mathrm{d}T(\tau)$ be average expenditure on variable inputs and average payment of the operating cost among those that pay the entry cost. In addition, let $\hat{s}^M$ be the aggregate share of measured expenditures spent on intermediate inputs. Per entrant, the average expenditure on capital can be expressed as $\hat{\alpha}(1-\hat{s}^M)V$, while the average expenditure on labor is $wf^E + O + (1-\hat{\alpha})(1-\hat{s}^M)V$. The underlying capital share is thus

$$\alpha = \frac{\hat{\alpha}(1-\hat{s}^M)V}{wf^E + O + (1-\hat{s}^M)V}$$

Free entry requires that $wf^E = \frac{1}{\varepsilon-1}V - O$. Together these yield

$$\alpha = \frac{(1-\hat{s}^M)}{\frac{1}{\varepsilon-1} + (1-\hat{s}^M)}\hat{\alpha} \tag{H.2}$$

In Web Appendix H.4 we show that $\hat{\sigma}^{agg}$ corresponds to our baseline estimate. We also show that differentiating (H.2) and rearranging yields

$$\sigma^{agg} - \hat{\sigma}^{agg} = \frac{\frac{\mathrm{d}\ln(1-\hat{s}^M)}{\mathrm{d}\ln w/r} + \hat{\alpha}(1-\hat{\sigma}^{agg})}{1 + (1-\hat{\alpha})(1-\hat{s}^M)(\varepsilon-1)}$$

We then estimate that $\sigma^{agg} - \hat{\sigma}^{agg}$ averages 0.072 across all industries in all years.[14] A small positive gap between σ^{agg} and $\hat{\sigma}^{agg}$ is in line with our Monte Carlo analysis described in Web Appendix H.5.

H.2 Proofs when Measured Inputs Include Overhead Costs

Define c^v_τ as unit cost of output, and let c^O_τ be the cost per unit of the overhead cost, so that the overhead cost for a plant with technology τ is $c^O_\tau f^O$. This notation allows for various assumptions about the factor content of overhead costs, covering the case in which overhead costs are denominated in final output ($c^O_\tau = P$), a plant's own output, ($c^O_\tau = c^v_\tau$) or labor ($c^O_\tau = w$). Thus plant τ's expenditure is $z_\tau = c^v_\tau y_\tau + c^O_\tau f^O$. $s^v_\tau \equiv \frac{c^v_\tau y_\tau}{z_\tau}$ is the share of τ's expenditure spent on variable costs. Then a plant's expenditure on capital and labor as a fraction of the aggregate expenditure is

$$\theta_\tau \equiv \frac{(1-s^M_\tau)z_\tau}{\int (1-s^M_{\tilde{\tau}})z_{\tilde{\tau}}E_{\tilde{\tau}}\mathrm{d}T(\tilde{\tau})}.$$

The aggregate capital share is $\alpha = \int \alpha_\tau \theta_\tau E_\tau \mathrm{d}T(\tau)$. Before deriving an expression for the aggregate elasticity of substitution, we begin with a lemma:

Lemma H.1

$$s^v_\tau \frac{\mathrm{d}\ln c_\tau/P}{\mathrm{d}\ln w/r} + (1-s^v_\tau)\frac{\mathrm{d}\ln c^O_\tau/P}{\mathrm{d}\ln w/r} = -(1-s^M_\tau)(\alpha_\tau - \alpha^M)$$

Proof. Let α^v_τ be the share of capital and labor used for variable costs that is spent on capital, and let s^{Mv}_τ be the share of variable costs spent on materials. Similarly, let α^O_τ be the share of capital

[14] Since the overhead cost is unmeasured, we have $\hat{\varepsilon} = \varepsilon$. We compute $\frac{\mathrm{d}\ln(1-\hat{s}^M)}{\mathrm{d}\ln w/r}$ directly after showing that it is closely related to the expression for ζ^N_n in Proposition 2. In most years it is slightly negative and is always close to zero.

and labor used for overhead costs that is spent on capital, and let s_τ^{MO} be the share of overhead costs spent on materials. Then Shephard's Lemma implies

$$\begin{aligned} s_\tau^v \frac{\mathrm{d}\ln c_\tau/P}{\mathrm{d}\ln w/P} + (1-s_\tau^v)\frac{\mathrm{d}\ln c_\tau^O/r}{\mathrm{d}\ln w/r} &= s_\tau^v\left[(1-s_\tau^{Mv})(1-\alpha_\tau^v) + s_\tau^{Mv}(1-\alpha^M) - (1-\alpha^M)\right] \\ &\quad +(1-s_\tau^v)\left[(1-s_\tau^{MO})(1-\alpha_\tau^O) + s_\tau^{MO}(1-\alpha^M) - (1-\alpha^M)\right] \\ &= (1-s_\tau^M)(1-\alpha_\tau) + s_\tau^M(1-\alpha^M) - (1-\alpha^M) \\ &= -(1-s_\tau^M)(\alpha_\tau - \alpha^M) \end{aligned}$$

■

With this in hand, we derive the following expression for the aggregate elasticity of substitution:

Proposition H.1 *If measured costs include overhead costs, the aggregate elasticity of substitution is*

$$\sigma^{agg} = (1-\chi)\bar{\sigma} + \int \frac{\alpha_\tau - \alpha}{\alpha(1-\alpha)}\theta_\tau \frac{\mathrm{d}E_\tau}{\mathrm{d}\ln w/r}\mathrm{d}T(\tau) + \chi(1-\bar{s}^M)\bar{\zeta} + \varepsilon\int \frac{\alpha_\tau - \alpha}{\alpha(1-\alpha)} s_\tau^v \frac{\mathrm{d}\ln P/c_\tau^v}{\mathrm{d}\ln w/r}\theta_\tau E_\tau \mathrm{d}T(\tau) \tag{H.3}$$

where σ_τ, $\bar{\sigma}$, χ, ζ_τ, $\bar{\zeta}$, and $\bar{s}^M$ are defined so that

$$\begin{aligned} \sigma_\tau - 1 &= \frac{\mathrm{d}\ln\frac{\alpha_\tau}{1-\alpha_\tau}}{\mathrm{d}\ln w/r} \\ \bar{\sigma} &= \frac{\int \alpha_\tau(1-\alpha_\tau)\theta_\tau E_\tau \sigma_\tau \mathrm{d}T(\tau)}{\int \alpha_\tau(1-\alpha_\tau)\theta_\tau E_\tau \mathrm{d}T(\tau)} \\ \chi &= \frac{\int (\alpha_\tau - \alpha)^2\theta_\tau E_\tau \mathrm{d}T(\tau)}{\alpha(1-\alpha)} \\ (\alpha^M - \alpha_\tau)(\zeta_\tau - 1) &= \frac{\mathrm{d}\ln\frac{s_\tau^M}{1-s_\tau^M}}{\mathrm{d}\ln w/r} \\ \bar{\zeta} &= \frac{\int (\alpha_\tau-\alpha)(\alpha_\tau-\alpha^M)s_\tau^M\theta_\tau E_\tau\zeta_\tau \mathrm{d}T(\tau)}{\int (\alpha_\tau-\alpha)(\alpha_\tau-\alpha^M)s_\tau^M\theta_\tau E_\tau \mathrm{d}T(\tau)} \\ \bar{s}^M &= \frac{\int (\alpha_\tau-\alpha)(\alpha_\tau-\alpha^M)\theta_\tau E_\tau s_\tau^M \mathrm{d}T(\tau)}{\int (\alpha_\tau-\alpha)(\alpha_\tau-\alpha^M)\theta_\tau E_\tau \mathrm{d}T(\tau)} \end{aligned}$$

Proof. Differentiating $\alpha = \int \alpha_\tau\theta_\tau E_\tau \mathrm{d}T(\tau)$ yields

$$\frac{\mathrm{d}\alpha}{\mathrm{d}\ln w/r} = \int \frac{\mathrm{d}\alpha_\tau}{\mathrm{d}\ln w/r}\theta_\tau E_\tau \mathrm{d}T(\tau) + \int (\alpha_\tau - \alpha)\theta_\tau\frac{\mathrm{d}E_\tau}{\mathrm{d}\ln w/r}\mathrm{d}T(\tau) + \int (\alpha_\tau-\alpha)\frac{\mathrm{d}\ln\theta_\tau}{\mathrm{d}\ln w/r}\theta_\tau E_\tau \mathrm{d}T(\tau) \tag{H.4}$$

Using $\sigma^{agg} - 1 = \frac{1}{\alpha(1-\alpha)}\frac{\mathrm{d}\alpha}{\mathrm{d}\ln w/r}$, $\sigma_\tau - 1 = \frac{1}{\alpha_\tau(1-\alpha_\tau)}\frac{\mathrm{d}\alpha_\tau}{\mathrm{d}\ln w/r}$, and the definition of $\bar{\sigma}$, we can express this as

$$\sigma^{agg} - 1 = (1-\chi)(\bar{\sigma}-1) + \int \frac{\alpha_\tau - \alpha}{\alpha(1-\alpha)}\theta_\tau\frac{\mathrm{d}E_\tau}{\mathrm{d}\ln w/r}\mathrm{d}T(\tau) + \int \frac{\alpha_\tau-\alpha}{\alpha(1-\alpha)}\frac{\mathrm{d}\ln\theta_\tau}{\mathrm{d}\ln w/r}\theta_\tau E_\tau \mathrm{d}T(\tau) \tag{H.5}$$

The last term can be decomposed into two parts:

$$\int (\alpha_\tau - \alpha)\frac{\mathrm{d}\ln\theta_\tau}{\mathrm{d}\ln w/r}\theta_\tau E_\tau \mathrm{d}T(\tau) = \int (\alpha_\tau - \alpha)\frac{\mathrm{d}\ln(1-s^M_\tau)}{\mathrm{d}\ln w/r}\theta_\tau E_\tau \mathrm{d}T(\tau) + \int (\alpha_\tau - \alpha)\frac{\mathrm{d}\ln z_\tau}{\mathrm{d}\ln w/r}\theta_\tau E_\tau \mathrm{d}T(\tau) \tag{H.6}$$

We tackle each of these two terms separately. For the first term, noting that $\frac{\mathrm{d}\ln(1-s^M_\tau)}{\mathrm{d}\ln w/r} = -s^M_\tau \frac{\mathrm{d}\ln \frac{s^M_\tau}{1-s^M_\tau}}{\mathrm{d}\ln w/r} = s^M_\tau(\alpha_\tau - \alpha^M)(\zeta_\tau - 1)$, the definitions of $\bar{\zeta}$, $\bar{s}^M$, and χ imply:

$$\begin{aligned}
\int (\alpha_\tau - \alpha)\frac{\mathrm{d}\ln(1-s^M_\tau)}{\mathrm{d}\ln w/r}\theta_\tau E_\tau \mathrm{d}T(\tau) &= \int (\alpha_\tau - \alpha)s^M_\tau(\alpha_\tau - \alpha^M)(\zeta_\tau - 1)\theta_\tau E_\tau \mathrm{d}T(\tau) \\
&= \int (\alpha_\tau - \alpha)s^M_\tau(\alpha_\tau - \alpha^M)\theta_\tau E_\tau \mathrm{d}T(\tau)(\bar{\zeta} - 1) \\
&= \int (\alpha_\tau - \alpha)(\alpha_\tau - \alpha^M)\theta_\tau E_\tau \mathrm{d}T(\tau)\bar{s}^M(\bar{\zeta} - 1) \\
&= \alpha(1-\alpha)\chi\bar{s}^M_\tau(\bar{\zeta} - 1)
\end{aligned}$$

Second, given the price it sets, a plant's expenditure on variable inputs is $c^v_\tau y_\tau = \left(\frac{\varepsilon}{\varepsilon-1}\right)^{-\varepsilon} Y P^\varepsilon (c^v_\tau)^{1-\varepsilon}$. With this, along with the fact that the definition of the elasticity of substitution holds Y constant, we have

$$\begin{aligned}
\frac{\mathrm{d}\ln z_\tau/P}{\mathrm{d}\ln w/r} &= s^v_\tau \frac{\mathrm{d}\ln Y(P/c^v_\tau)^{\varepsilon-1}}{\mathrm{d}\ln w/r} + (1-s^v_\tau)\frac{\mathrm{d}\ln(c^O_\tau/P)}{\mathrm{d}\ln w/r} \\
&= \varepsilon s^v_\tau \frac{\mathrm{d}\ln P/c^v_\tau}{\mathrm{d}\ln w/r} + s^v_\tau \frac{\mathrm{d}\ln c^v_\tau/P}{\mathrm{d}\ln w/r} + (1-s^v_\tau)\frac{\mathrm{d}\ln(c^O_\tau/P)}{\mathrm{d}\ln w/r} \\
&= \varepsilon s^v_\tau \frac{\mathrm{d}\ln P/c^v_\tau}{\mathrm{d}\ln w/r} - (1-s^M_\tau)(\alpha_\tau - \alpha^M)
\end{aligned}$$

where the last line follows from Lemma H.1. Thus using the definition of $\bar{s}^M$ we have

$$\int \frac{\alpha_\tau - \alpha}{\alpha(1-\alpha)}\frac{\mathrm{d}\ln z_\tau}{\mathrm{d}\ln w/r}\theta_\tau E_\tau \mathrm{d}T(\tau) = \varepsilon \int \frac{\alpha_\tau - \alpha}{\alpha(1-\alpha)} s^v_\tau \frac{\mathrm{d}\ln P/c^v_\tau}{\mathrm{d}\ln w/r}\theta_\tau E_\tau \mathrm{d}T(\tau) - \chi(1-\bar{s}^M) \tag{H.7}$$

The proposition follows from adding one to each side. ■

We now specialize to to various assumptions about the factor content of entry and overhead costs.

Corollary 1 *Suppose that overhead costs are denominated in common units across plants, so that* $c^O_\tau = c^O$*. Then*

$$\int \frac{\alpha_\tau - \alpha}{\alpha(1-\alpha)} s^v_\tau \frac{\mathrm{d}\ln P/c^v_\tau}{\mathrm{d}\ln w/r}\theta_\tau E_\tau \mathrm{d}T(\tau) = \chi(1-\bar{s}^M) + \frac{\mathrm{d}\ln c^O/P}{\mathrm{d}\ln w/r}\frac{1-s^v}{1-s^M}\frac{\int(\alpha_\tau - \alpha)(1-s^M_\tau)E_\tau \mathrm{d}T(\tau)}{\alpha(1-\alpha)\int E_\tau \mathrm{d}T(\tau)}$$

Proof. Lemma H.1 can be rearranged as $s^v_\tau \frac{\mathrm{d}\ln P/c^v_\tau}{\mathrm{d}\ln w/r} = (1-s^M_\tau)(\alpha_\tau - \alpha) + (1-s^v_\tau)\frac{\mathrm{d}\ln c^O/P}{\mathrm{d}\ln w/r}$. The integral can be expressed as

$$\int (\alpha_\tau - \alpha)s^v_\tau \frac{\mathrm{d}\ln P/c^v_\tau}{\mathrm{d}\ln w/r}\theta_\tau E_\tau \mathrm{d}T(\tau) = \chi\alpha(1-\alpha)(1-\bar{s}^M) + \int (\alpha_\tau - \alpha)(1-s^v_\tau)\frac{\mathrm{d}\ln c^O/P}{\mathrm{d}\ln w/r}\theta_\tau E_\tau \mathrm{d}T(\tau)$$

The second term can be simplified by factoring out $\frac{\mathrm{d}\ln c^O/P}{\mathrm{d}\ln w/r}$ and using $(1-s^v_\tau)\theta_\tau = \frac{(1-s^M_\tau)(1-s^v_\tau)z_\tau}{(1-s^M)\int z_{\tilde{\tau}}E_{\tilde{\tau}}\mathrm{d}T(\tau)} = \frac{(1-s^M_\tau)c^O f^O}{(1-s^M)\int z_{\tilde{\tau}}E_{\tilde{\tau}}\mathrm{d}T(\tau)}$:

$$
\begin{aligned}
\int(\alpha_\tau-\alpha)(1-s^v_\tau)\theta_\tau E_\tau \mathrm{d}T(\tau) &= \frac{c^O f^O}{(1-s^M)\int z_{\tilde{\tau}}E_{\tilde{\tau}}\mathrm{d}T(\tau)}\int(\alpha_\tau-\alpha)(1-s^M_\tau)E_\tau\mathrm{d}T(\tau)\\
&= \frac{c^O f^O\int E_\tau\mathrm{d}T(\tau)}{(1-s^M)\int z_{\tilde{\tau}}E_{\tilde{\tau}}\mathrm{d}T(\tau)}\frac{\int(\alpha_\tau-\alpha)(1-s^M_\tau)E_\tau\mathrm{d}T(\tau)}{\int E_\tau\mathrm{d}T(\tau)}\\
&= \frac{(1-s^v)}{(1-s^M)}\frac{\int(\alpha_\tau-\alpha)(1-s^M_\tau)E_\tau\mathrm{d}T(\tau)}{\int E_\tau\mathrm{d}T(\tau)}
\end{aligned}
$$

■

Finally, we show that if entry costs are denominated in final output, then $\alpha^M \equiv \frac{\mathrm{d}\ln P/w}{\mathrm{d}\ln w/r} = \alpha$, as in the baseline model.

Proposition H.2 *Suppose that entry costs are denominated in final output. Then* $\alpha^M \equiv \frac{\mathrm{d}\ln P/w}{\mathrm{d}\ln w/r} = \alpha$.

Proof. Since $y_\tau = c^v_\tau Y P^\varepsilon\left(\frac{\varepsilon}{\varepsilon-1}c^v_\tau\right)^{-\varepsilon}$, free entry implies

$$
\begin{aligned}
Pf^E &= \int \max\left\{\frac{1}{\varepsilon-1}c^v_\tau y_\tau - c^O_\tau f^O, 0\right\}E_\tau\mathrm{d}T(\tau)\\
&= \int \max\left\{\frac{1}{\varepsilon-1}c^v_\tau Y P^\varepsilon\left(\frac{\varepsilon}{\varepsilon-1}c^v_\tau\right)^{-\varepsilon} - c^O_\tau f^O, 0\right\}E_\tau\mathrm{d}T(\tau)
\end{aligned}
$$

Dividing by P, differentiating with respect to w/r, and using the fact that $\frac{\mathrm{d}E_\tau}{\mathrm{d}\ln w/r}\neq 0$ implies that the plant is at the margin of entering so that $\frac{1}{\varepsilon-1}c^v_\tau Y P^\varepsilon\left(\frac{\varepsilon}{\varepsilon-1}c^v_\tau\right)^{-\varepsilon} - c^O_\tau f^O = 0$, gives

$$
\begin{aligned}
0 &= \int\left[\frac{1}{\varepsilon-1}\left(\frac{\varepsilon}{\varepsilon-1}\right)^{-\varepsilon}Y\left(P/c^v_\tau\right)^{\varepsilon-1}\frac{\mathrm{d}\ln Y\left(P/c^v_\tau\right)^{\varepsilon-1}}{\mathrm{d}\ln w/r} - \frac{c^O_\tau}{P}f^O\frac{\mathrm{d}\ln c^O_\tau/P}{\mathrm{d}\ln w/r}\right]E_\tau\mathrm{d}T(\tau)\\
&= \int\left[\frac{1}{\varepsilon-1}s^v_\tau\frac{\mathrm{d}\ln Y\left(P/c^v_\tau\right)^{\varepsilon-1}}{\mathrm{d}\ln w/r} - (1-s^v_\tau)\frac{\mathrm{d}\ln c^O_\tau/P}{\mathrm{d}\ln w/r}\right]\frac{z_\tau}{P}E_\tau\mathrm{d}T(\tau)
\end{aligned}
$$

Since the elasticity holds Y constant, we can rearrange this and use Lemma H.1 to get

$$
\begin{aligned}
0 &= \frac{\int\left[s^v_\tau\frac{\mathrm{d}\ln c^v_\tau/P}{\mathrm{d}\ln w/r} + (1-s^v_\tau)\frac{\mathrm{d}\ln c^O_\tau/P}{\mathrm{d}\ln w/r}\right]z_\tau E_\tau\mathrm{d}T(\tau)}{-\int(1-s^M_\tau)z_\tau E_\tau\mathrm{d}T(\tau)}\\
&= \frac{\int\left[-(1-s^M_\tau)(\alpha_\tau-\alpha^M)\right]z_\tau E_\tau\mathrm{d}T(\tau)}{-\int(1-s^M_\tau)z_\tau E_\tau\mathrm{d}T(\tau)}\\
&= \int(\alpha_\tau-\alpha^M)\theta_\tau E_\tau\mathrm{d}T(\tau)\\
&= \alpha-\alpha^M
\end{aligned}
$$

■

Corollary 2 *Suppose that overhead costs use final output. Then*

$$\sigma^{agg} = (1-\chi)\bar{\sigma} + \int \frac{\alpha_\tau - \alpha}{\alpha(1-\alpha)}\theta_\tau \frac{\mathrm{d}E_\tau}{\mathrm{d}\ln w/r}\mathrm{d}T(\tau) + \chi(1-\bar{s}^M)\bar{\zeta} + \chi(1-\bar{s}^M)\varepsilon$$

Proof. This follows from noting that if the overhead cost is denominated in final output, $\frac{\mathrm{d}\ln c^O/P}{\mathrm{d}\ln w/r} = 0$. ■

Corollary 3 *Suppose that overhead costs use labor and entry costs use final output. Then*

$$\begin{aligned}\sigma^{agg} &= (1-\chi)\bar{\sigma} + \int \frac{\alpha_\tau - \alpha}{\alpha(1-\alpha)}\theta_\tau \frac{\mathrm{d}E_\tau}{\mathrm{d}\ln w/r}\mathrm{d}T(\tau) + \chi(1-\bar{s}^M)\bar{\zeta} + \chi(1-\bar{s}^M)\varepsilon \\ &+\varepsilon\frac{1-s^v}{(1-\alpha)(1-s^M)}\frac{\int(\alpha_\tau-\alpha)(1-s_\tau^M)E_\tau \mathrm{d}T(\tau)}{\int E_\tau \mathrm{d}T(\tau)} \end{aligned} \tag{H.8}$$

Proof. This follows from noting that if the overhead cost is denominated in labor so that $c^O = w$, then $\frac{\mathrm{d}\ln c^O/P}{\mathrm{d}\ln w/r} = 1-(1-\alpha^M) = \alpha$. ■

For the case in which overhead costs use labor, we can bound the extra term in (H.8). First, note that since entry cost are in labor, $\frac{1-s^v}{(1-\alpha)(1-s^M)} \in [0,1]$ because the numerator is the share of total expenditure spent on overhead costs and the denominator is the share of total expenditure on labor spent on labor. Second, in our data, α_τ and $1-s_\tau^M$ are uncorrelated, so we can express this as $\frac{\int(\alpha_\tau-\alpha)(1-s_\tau^M)E_\tau \mathrm{d}T(\tau)}{\int E_\tau \mathrm{d}T(\tau)} = \left(\frac{\int \alpha_\tau E_\tau \mathrm{d}T(\tau)}{\int E_\tau \mathrm{d}T(\tau)} - \alpha\right)\left(1 - \frac{\int s_\tau^M E_\tau \mathrm{d}T(\tau)}{\int E_\tau \mathrm{d}T(\tau)}\right)$. Since larger plants tend to be more capital intensive, this is negative. However given the magnitude of the difference between the average and aggregate capital shares and the magnitude of materials shares, this term is quantitatively small.

Corollary 4 *Suppose that overhead costs use a plant's own output. Then*

$$\sigma^{agg} = (1-\chi)\bar{\sigma} + \int \frac{\alpha_\tau - \alpha}{\alpha(1-\alpha)}\theta_\tau \frac{\mathrm{d}E_\tau}{\mathrm{d}\ln w/r}\mathrm{d}T(\tau) + \chi(1-\bar{s}^M)\bar{\zeta} + \chi(1-\bar{s}^M)\bar{s}^v\varepsilon$$

where $\bar{s}^v \equiv \frac{\int(\alpha_\tau-\alpha)(\alpha_\tau-\alpha^M)(1-s_\tau^M)\theta_\tau E_\tau s_\tau^v \mathrm{d}T(\tau)}{\int(\alpha_\tau-\alpha)(\alpha_\tau-\alpha^M)(1-s_\tau^M)\theta_\tau E_\tau \mathrm{d}T(\tau)}$ *is a weighted average of plants' shares of variable costs. In addition, define* $\hat{\varepsilon}_\tau$ *so that plant* τ*'s ratio of revenue to cost is* $\frac{\hat{\varepsilon}_\tau}{\hat{\varepsilon}_\tau-1} = \frac{p_\tau y_\tau}{c_\tau^v y_\tau + c_\tau^O f^O}$. *Then* $s_\tau^v\varepsilon \le \hat{\varepsilon}_\tau$.

Proof. If overhead costs use a plant's own output, then

$$\frac{\mathrm{d}\ln c_\tau^v/P}{\mathrm{d}lnw/r} = \frac{\mathrm{d}\ln c_\tau^O/P}{\mathrm{d}lnw/r} = (1-s_\tau^M)(1-\alpha_\tau) + s_\tau^M(1-\alpha^M) - (1-\alpha^M) = -(1-s_\tau^M)(\alpha_\tau - \alpha^M)$$

The results follows from plugging this into (H.3) and using the definition of $\bar{s}^v$. In addition plant τ's ratio of revenue to cost is

$$\frac{\hat{\varepsilon}_\tau}{\hat{\varepsilon}_\tau - 1} = \frac{\frac{\varepsilon}{\varepsilon-1}c_\tau^v y_\tau}{c_\tau^v y_\tau + c_\tau^O f^O} = \frac{\varepsilon}{\varepsilon-1}s_\tau^v,$$

This can be rearranged as

$$\hat{\varepsilon}_\tau = \frac{\frac{\varepsilon}{\varepsilon-1}s_\tau^v}{\frac{\varepsilon}{\varepsilon-1}s_\tau^v - 1} = \frac{\varepsilon s_\tau^v}{1-\varepsilon\left(1-s_\tau^v\right)} > \varepsilon s_\tau^v$$

■

H.3 Interpreting Cross-sectional Estimates

This section shows both analytically and using a Monte Carlo simulation that, to a first order, our baseline estimate of the micro elasticity of substitution captures changes in the plant-level capital shares coming from both the intensive and extensive margins of adjustment: within-plant substitution and changes from entry and exit. This allows us to connect our baseline estimate to the first two terms of (H.1):

$$\int \frac{\mathrm{d}\alpha_\tau}{\mathrm{d}\ln w/r}\theta_\tau E_\tau \mathrm{d}T(\tau) + \int \frac{\mathrm{d}E_\tau}{\mathrm{d}\ln w/r}(\alpha_\tau - \alpha)\theta_\tau dT(\tau) \approx \alpha(1-\alpha)(1-\chi)(\hat{\bar{\sigma}}-1),$$

where $\hat{\bar{\sigma}}$ is derived from the estimate of the cross-sectional micro elasticity (C.1).

Consider next estimating (C.4) using OLS in an environment with entry and exit. Let $\tilde{I}$ be the set of potential entrants: those that pay the entry cost, whether or not they pay the fixed operating cost. The OLS estimator yields

$$\hat{\beta}_E = \frac{\sum_{i\in\tilde{I}}\left(\omega_{cz(i)}-\bar{\omega}\right)E_i\left(\alpha_i-\bar{\alpha}\right)}{\sum_{i\in\tilde{I}}\left(\omega_{cz(i)}-\bar{\omega}\right)^2 E_i} = \frac{\sum_{i\in\tilde{I}}\left(\omega_{cz(i)}-\bar{\omega}\right)E_i\left(\alpha_i-\alpha\right)}{\sum_{i\in\tilde{I}}\left(\omega_{cz(i)}-\bar{\omega}\right)^2 E_i}$$

where the constants $\bar{\alpha} \equiv \frac{\sum_{i\in\tilde{I}} E_i\alpha_i}{\sum_{i\in\tilde{I}} E_i}$ and $\bar{\omega} \equiv \frac{\sum_{i\in\tilde{I}} E_i\omega_{cz(i)}}{\sum_{i\in\tilde{I}} E_i}$.

Consider the function $\alpha_i(\omega)$ and $E_i(\omega)$, which are what i's capital share and operating status would be with relative factor prices ω so that, abusing notation, $\alpha_i = \alpha_i\left(\omega_{cz(i)}\right)$ and $E_i = E_i(\omega_{cz(i)})$. A first order approximation of $E_i(\bar{\omega})\left(\alpha_i(\bar{\omega})-\alpha\right)$ around $\omega_{cz(i)}$ yields

$$\begin{aligned} E_i\left(\alpha_i-\alpha\right) =& E_i(\omega_{cz(i)})\left(\alpha_i\left(\omega_{cz(i)}\right)-\alpha\right) \\ \approx& E_i(\bar{\omega})\left(\alpha_i(\bar{\omega})-\alpha\right) + E_i\frac{\mathrm{d}\alpha_i(\omega_{cz(i)})}{\mathrm{d}\omega}(\omega_{cz(i)}-\bar{\omega}) \\ &+ \frac{\mathrm{d}E_i(\omega_{cz(i)})}{\mathrm{d}\omega}(\alpha_i-\alpha)(\omega_{cz(i)}-\bar{\omega}) + O\left(\left(\omega_{cz(i)}-\bar{\omega}\right)^2\right) \end{aligned}$$

Combining these equations and rearranging yields

$$\begin{aligned} \hat{\beta}_E \approx& \sum_{i\in\tilde{I}}\theta_i E_i\frac{\mathrm{d}\alpha_i}{\mathrm{d}\omega} + \sum_{i\in\tilde{I}}\theta_i\frac{\mathrm{d}E_i}{\mathrm{d}\omega}(\alpha_i-\alpha) \\ &+ \sum_{i\in\tilde{I}}\left(\frac{(\omega_{cz(i)}-\bar{\omega})^2}{\sum_{\tilde{i}\in\tilde{I}}(\omega_{cz(\tilde{i})}-\bar{\omega})^2 E_{\tilde{i}}} - \theta_i\right)\left(\frac{\mathrm{d}E_i}{\mathrm{d}\omega}(\alpha_i-\alpha) + E_i\frac{\mathrm{d}\alpha_i}{\mathrm{d}\omega}\right) \\ &+ \frac{\sum_i\left(\omega_{cz(i)}-\bar{\omega}\right)E_i\left(\alpha_i(\bar{\omega})-\alpha\right)}{\sum_i\left(\omega_{cz(i)}-\bar{\omega}\right)^2 E_i} \end{aligned}$$

The first two terms are exactly the terms needed to use in the formula to recover the aggregate elasticity. The next two terms represent potential sources of bias. The third term reflects the fact that weights on each observation used by OLS are different from the ones needed for aggregation. As discussed above in Web Appendix C.4 (and as we confirm in the Monte Carlo below) the bias from this term is negligible. We now argue that the final term is likely to be positive, leading to an upward bias, i.e., the estimate will tend to overstate $\bar{\sigma}$. The argument is almost identical to the one discussed above in Web Appendix C.4 when we studied a regression of capital shares on local factor prices weighting by θ_i. Here, we instead weight by E_i. Since E_i is monotonically related to

θ_i, the argument is the same. Recall that larger plants tend to have higher capital shares, even within narrowly defined industries. Thus we expect $\sum E_i \left(\alpha_i\left(\bar{\omega}\right) - \bar{\alpha}\right) > 0$. An increase in ω raises E_i more when $\alpha_i(\bar{\omega})$ is larger, i.e., when $\left(\alpha_i\left(\bar{\omega}\right) - \alpha\right)$ is larger. Thus the positive covariance would be strengthened. Conversely, a reduction in ω weakens the covariance. Together, these imply that the final term is likely to be positive.

H.4 Foregone Labor

Claim H.1 *Our baseline estimate corresponds to* $\hat{\sigma}^{agg}$*:*

$$\hat{\sigma}^{agg} = (1-\hat{\chi})\hat{\bar{\sigma}} + \hat{\chi}\left[\hat{\bar{s}}^M\hat{\bar{\zeta}} + (1-\hat{\bar{s}}^M_\tau)\hat{\varepsilon}\right]$$

Proof. A plant's measured expenditure on capital and labor as a fraction of the aggregate measured expenditure is

$$\hat{\theta}_\tau \equiv \frac{(1-\hat{s}^M_\tau)c^v_\tau y_\tau}{\int(1-\hat{s}^M_{\tilde{\tau}})c^v_{\tilde{\tau}} y_{\tilde{\tau}} E_{\tilde{\tau}} \mathrm{d}T(\tilde{\tau})}.$$

The measured aggregate capital share is $\hat{\alpha} = \int \hat{\alpha}_\tau \hat{\theta}_\tau E_\tau \mathrm{d}T(\tau)$ and, as in our baseline, differentiating yields

$$\frac{\mathrm{d}\hat{\alpha}}{\mathrm{d}\ln w/r} = \int \frac{\mathrm{d}\hat{\alpha}_\tau}{\mathrm{d}\ln w/r}\hat{\theta}_\tau E_\tau \mathrm{d}T(\tau) + \int (\hat{\alpha}_\tau - \hat{\alpha})\hat{\theta}_\tau \frac{\mathrm{d}E_\tau}{\mathrm{d}\ln w/r}\mathrm{d}T(\tau) + \int (\hat{\alpha}_\tau - \hat{\alpha})\frac{\mathrm{d}\ln \hat{\theta}_\tau}{\mathrm{d}\ln w/r}\hat{\theta}_\tau E_\tau \mathrm{d}T(\tau)$$

By definition, $\hat{\sigma}^{agg} - 1 = \frac{1}{\hat{\alpha}(1-\hat{\alpha})}\frac{\mathrm{d}\hat{\alpha}}{\mathrm{d}\ln w/r}$. As in the last section, the sum of the first two terms is $\int \frac{\mathrm{d}\hat{\alpha}_\tau}{\mathrm{d}\ln w/r}\hat{\theta}_\tau E_\tau \mathrm{d}T(\tau) + \int (\hat{\alpha}_\tau - \hat{\alpha})\hat{\theta}_\tau \frac{\mathrm{d}E_\tau}{\mathrm{d}\ln w/r}\mathrm{d}T(\tau) = \hat{\alpha}(1-\hat{\alpha})(1-\hat{\chi})(\hat{\bar{\sigma}}-1)$. Therefore it remains only to show that $\int (\hat{\alpha}_\tau - \hat{\alpha})\frac{\mathrm{d}\ln \hat{\theta}_\tau}{\mathrm{d}\ln w/r}\hat{\theta}_\tau E_\tau \mathrm{d}T(\tau) = \hat{\alpha}(1-\hat{\alpha})\hat{\chi}\left[\hat{\bar{s}}^M(\hat{\bar{\zeta}}-1) + (1-\hat{\bar{s}}^M)(\hat{\varepsilon}-1)\right]$. Note that

$$\int (\hat{\alpha}_\tau - \hat{\alpha})\frac{\mathrm{d}\ln \hat{\theta}_\tau}{\mathrm{d}\ln w/r}\hat{\theta}_\tau E_\tau \mathrm{d}T(\tau) = \int (\hat{\alpha}_\tau - \hat{\alpha})\frac{\mathrm{d}\ln(1-\hat{s}^M_\tau)}{\mathrm{d}\ln w/r}\hat{\theta}_\tau E_\tau \mathrm{d}T(\tau) + \int (\hat{\alpha}_\tau - \hat{\alpha})\frac{\mathrm{d}\ln c^v_\tau y_\tau}{\mathrm{d}\ln w/r}\hat{\theta}_\tau E_\tau \mathrm{d}T(\tau)$$

The argument that $\int (\hat{\alpha}_\tau - \hat{\alpha})\frac{\mathrm{d}\ln(1-\hat{s}^M_\tau)}{\mathrm{d}\ln w/r}\hat{\theta}_\tau E_\tau \mathrm{d}T(\tau) = \hat{\alpha}(1-\hat{\alpha})\hat{\chi}\hat{\bar{s}}^M(\hat{\bar{\zeta}}-1)$ is exactly the same as in the previous section. Since $c^v_\tau y_\tau = c^v_\tau Y P^\varepsilon \left(\frac{\varepsilon}{\varepsilon-1}c^v_\tau\right)^{-\varepsilon}$, we have

$$\begin{aligned}
\int (\hat{\alpha}_\tau - \hat{\alpha})\frac{\mathrm{d}\ln c^v_\tau y_\tau}{\mathrm{d}\ln w/r}\hat{\theta}_\tau E_\tau \mathrm{d}T(\tau) &= \int (\hat{\alpha}_\tau - \hat{\alpha})(1-\varepsilon)\frac{\mathrm{d}\ln c^v_\tau/r}{\mathrm{d}\ln w/r}\hat{\theta}_\tau E_\tau \mathrm{d}T(\tau) \\
&= (1-\varepsilon)\int (\hat{\alpha}_\tau - \hat{\alpha})\left[(1-\hat{s}^M_\tau)(1-\hat{\alpha}_\tau) + \hat{s}^M_\tau(1-\alpha^M)\right]\hat{\theta}_\tau E_\tau \mathrm{d}T(\tau) \\
&= (\varepsilon-1)\int (\hat{\alpha}_\tau - \hat{\alpha})\left[(1-\hat{s}^M_\tau)(\hat{\alpha}_\tau - \alpha^M)\right]\hat{\theta}_\tau E_\tau \mathrm{d}T(\tau) \\
&= (\varepsilon-1)(1-\hat{\bar{s}}^M_\tau)\hat{\chi}
\end{aligned}$$

Lastly, since overhead costs are not observed, $\frac{\hat{\varepsilon}}{\hat{\varepsilon}-1} = \frac{p_\tau y_\tau}{c_\tau y_\tau} = \frac{\varepsilon}{\varepsilon-1}$. ∎

Claim H.2

$$\sigma^{agg} - \hat{\sigma}^{agg} = \frac{\frac{d\ln(1-\hat{s}^M)}{d\ln w/r} + \hat{\alpha}(1-\hat{\sigma}^{agg})}{1+(1-\hat{\alpha})(1-\hat{s}^M)(\varepsilon-1)}$$

Proof. Define $B \equiv \frac{1-\hat{s}^M}{\frac{1}{\varepsilon-1}+(1-\hat{s}^M)}$ so that $\alpha = B\hat{\alpha}$. Taking logs and differentiating, then using $\sigma^{agg} - 1 = \frac{1}{\alpha(1-\alpha)}\frac{\mathrm{d}\alpha}{\mathrm{d}\ln w/r}$ and $\hat{\sigma}^{agg} - 1 = \frac{1}{\hat{\alpha}(1-\hat{\alpha})}\frac{\mathrm{d}\hat{\alpha}}{\mathrm{d}\ln w/r}$ yields

$$\frac{1}{\alpha}\frac{\mathrm{d}\alpha}{\mathrm{d}\ln w/r} = \frac{\mathrm{d}\ln B}{\mathrm{d}\ln w/r} + \frac{1}{\hat{\alpha}}\frac{\mathrm{d}\hat{\alpha}}{\mathrm{d}\ln w/r}$$
$$(1-\alpha)(\sigma^{agg}-1) = \frac{\mathrm{d}\ln B}{\mathrm{d}\ln w/r} + (1-\hat{\alpha})(\hat{\sigma}^{agg}-1)$$

This can be rearranged as

$$\sigma^{agg} - \hat{\sigma}^{agg} = \frac{1}{1-\alpha}\frac{\mathrm{d}\ln B}{\mathrm{d}\ln w/r} + \frac{\hat{\alpha}-\alpha}{1-\alpha}\left(1-\hat{\sigma}^{agg}\right)$$
$$= \frac{1-B}{1-\alpha}\left(\frac{d\ln\frac{B}{1-B}}{d\ln w/r} + \hat{\alpha}\left(1-\hat{\sigma}^{agg}\right)\right)$$

$\frac{B}{1-B} = (\varepsilon-1)(1-\hat{s}^M)$ implies $\frac{d\ln\frac{B}{1-B}}{d\ln w/r} = \frac{d\ln(1-\hat{s}^M)}{d\ln w/r}$. To complete the proof, we have

$$\frac{1-\alpha}{1-B} = \frac{1-B\hat{\alpha}}{1-B} = \frac{1-\frac{(1-\hat{s}^M)}{\frac{1}{\varepsilon-1}+(1-\hat{s}^M)}\hat{\alpha}}{1-\frac{(1-\hat{s}^M)}{\frac{1}{\varepsilon-1}+(1-\hat{s}^M)}} = 1+(1-\hat{\alpha})\left(1-\hat{s}^M\right)(\varepsilon-1)$$

■

H.5 Monte Carlo

We next confirm these arguments using a Monte Carlo exercise. We simulate an economy with 700 locations that each contain 100 plants. We normalize the rental rate to 1 and draw the natural log of each location's wage from a uniform (0,1) distribution. We assume that each plant produces using the CES production technology

$$Y_i = \frac{1}{\beta^\beta(1-\beta)^{1-\beta}}\left[\rho^{\frac{1}{\phi}}\left(A_iK\right)^{\frac{\phi-1}{\phi}} + (1-\rho)^{\frac{1}{\phi}}\left(B_iL\right)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}\beta} M^{1-\beta}$$

with an elasticity of substitution between capital and labor of 0.35, returns to scale ε of 4, and capital-labor aggregate output elasticity β of 0.3. We also draw technology parameters A_i and B_i from a joint lognormal. We normalize the mean of A_i to 1, and choose the mean of B_i, the variances of A_i and B_i as well as their covariance to match the following four moments: an aggregate capital share of 0.3, a value of χ of 0.1, the 90-10 ratio of marginal cost across plants to 2.7, and the coefficient of a regression of $\log(\frac{\alpha_i}{1-\alpha_i})$ on $\log\theta_i$ (weighting by θ_i) of 0.08.[15]

We examine two parameterizations of the entry cost and overhead operating cost; in the first, both the entry cost and overhead cost are paid in materials with materials price P. In the second,

[15] Figure 5 depicts the aggregate share for the manufacturing sector over time, and Figure 1 values of χ across industries. Table 1 in Syverson (2004) examines dispersion in productivity (our value corresponds to the 90-10 ratio in TFP computed using plant specific input elasticities). Table 3 in Raval (2019) reports the coefficient of regressions of the capital share to labor share ratio on value added, weighting by value added, with estimates ranging from 0.05 to 0.09 using the Census of Manufactures across years, and 0.06 to 0.11 using the Annual Survey of Manufactures.

both the entry cost and fixed cost are paid in unobserved local labor. For both sets of simulations, the ratio of the entry cost to the operating cost matters, but not their levels.

All firms pay the entry cost, and operate if their sales net of variable costs cover the operating cost. Given free entry, expected profits from entering are zero.

We then run 100 simulations for each parameterization of the entry cost and operating cost, for a ratio of the entry cost to operating cost of 2.5 to 1, 5 to 1, and 10 to 1. Table H.1 contains these estimates. Column (2) reports the true aggregate elasticity σ^{agg}: we simulate all firms' actions after increasing or decreasing wages in all localities by 5%, and compute the aggregate elasticity by finite difference. Columns (3) and (4) report the estimated micro elasticity $\hat{\sigma}$ and estimated aggregate elasticity $\hat{\sigma}^{agg}$ using our baseline cross-sectional regression approach and our aggregation framework. Column (5) reports an alternative estimate of the aggregate elasticity computed using a cross-sectional regression across locations where the dependent variable is the aggregate capital share in each location.

Using the parameterization where both fixed costs are in materials, our baseline estimate of the aggregate elasticity overstates the true aggregate elasticity by, on average, 0.15. Our baseline estimate of the micro elasticity is substantially above the true micro elasticity, and typically only slightly lower than the true aggregate elasticity.

Using the parameterization where both fixed costs are in unobserved labor, our baseline estimate of the aggregate elasticity also overstates the true aggregate elasticity, although the bias is smaller when operating costs shrink relative to entry costs. For example, the baseline estimate of the aggregate elasticity is about 0.3 larger than the true elasticity when entry costs are 2.5 times operating costs, but only 0.1 larger when entry costs are 10 times operating costs.

For both parameterizations, the alternative estimate of the aggregate elasticity from a cross-sectional regression of aggregate capital share from each location on the local wage is very close to the true aggregate elasticity. This specification mirrors those estimated in Appendix E.2; in that section, we used this alternative approach for each industry (4 digit SIC or 6 digit NAICS level) and found that the resulting estimates were fairly close to the average industry elasticity using of our baseline aggregation approach.

I Adjustment Frictions and Distortions

Section 2 showed that the relative importance of within-plant substitution and reallocation depends upon the variation in cost shares of capital. In that environment, this variation came from non-neutral differences in technology. On the other hand, as the recent misallocation literature emphasizes, some of this heterogeneity may be due to adjustment costs or other distortions. What are the implications for the aggregate elasticity of substitution if differences in capital shares came from distortions?

In Web Appendix I.1 we summarize our overall approach to characterizing the aggregate elasticity in environments with misallocation, and summarize our findings. The remainder of the section provides more detailed analytical and quantitative results. Web Appendix I.2 derives a general expression for the aggregate elasticity. Web Appendix I.3 studies exogenous distortions inspired by Hsieh and Klenow (2009). Web Appendix I.4 and Web Appendix I.5 provide analytical results for capital adjustment frictions, and Web Appendix I.6 uses Monte Carlo simulations to gauge the magnitude of the bias of our baseline estimates.

Table H.1 Monte Carlo Estimates of Substitution Elasticity with Entry

Ratio, Entry to Operating Cost	σ^{agg}	$\hat{\sigma}$	$\hat{\sigma}^{agg}$	Aggregate Regression
Entry and Operating Costs in Materials				
2.5 to 1	0.56	0.57	0.72	0.58
5 to 1	0.60	0.57	0.75	0.60
10 to 1	0.63	0.56	0.77	0.62
Entry and Operating Costs in Unobserved Labor				
2.5 to 1	0.60	0.84	0.93	0.63
5 to 1	0.60	0.69	0.79	0.55
10 to 1	0.60	0.57	0.69	0.51

Note: The table contains six specifications based on 100 simulations with two different parameterizations of entry and operating costs – in terms of materials or unobserved labor – and three values of the ratio of entry cost to operating cost: 2.5 to 1, 5 to 1, and 10 to 1. For each, we report the true aggregate elasticity by resolving all plants' actions in response to a 5% nationwide increase and 5% nationwide decrease in wages and estimating the elasticity through the finite difference (σ^{agg}); our baseline estimates of the micro elasticity from a regression of plants' log of the ratio of capital cost to labor cost on the log local wage ($\hat{\sigma}$); our baseline estimates of the aggregate elasticity using the estimated micro elasticity and our baseline formula ($\hat{\sigma}^{agg}$); and estimates of the aggregate elasticity using a regression of each location's log of the ratio of aggregate capital cost to aggregate labor cost on the log local wage.

I.1 Approach and Findings

General Framework To answer this question, we first characterize the aggregate elasticity in terms of how plants change their input expenditures in response to permanent changes in factor prices. Unlike Section 2, we do not assume plants choose inputs solely to minimize their cost.

To examine how each plant's input use would change with factor prices, we identify each plant with a history of shocks, h, which include shocks to demand and productivity. Let $H(h)$ be the distribution of histories, so the aggregate capital share is $\alpha = \int \alpha_h \theta_h dH(h)$. We define σ_h and ζ_h as the local response of plant h's relative factor expenditures to a change in factor prices, so $\sigma_h - 1 \equiv \frac{\mathrm{d}\ln \frac{\alpha_h}{1-\alpha_h}}{\mathrm{d}\ln w/r}$ and $(\alpha^M - \alpha_h)(\zeta_h - 1) = \frac{\mathrm{d}\ln \frac{s_h^M}{1-s_h^M}}{\mathrm{d}\ln w/r}$. We make no assumption about how these objects are related to h's production function; σ_h and ζ_h simply reflect how h's choices would change with different factor prices.

Following exactly the steps of Section 2.1, differentiating each side of $\alpha = \int \alpha_h \theta_h dH(h) = \int \alpha_h \frac{1-s_h^M}{1-s^M} \frac{z_h}{z} dH(h)$ for $\frac{z_h}{z} \equiv \frac{rK_h + wL_h + qM_h}{rK + wL + qM}$ and rearranging yields

$$\sigma^{agg} - 1 \quad = \quad (1-\chi)(\bar{\sigma} - 1) + \chi \bar{s}^M (\bar{\zeta} - 1) + \int_h \frac{(\alpha_h - \alpha)\theta_h}{\alpha(1-\alpha)} \frac{\mathrm{d}\ln z_h/z}{\mathrm{d}\ln w/r} dH(h)$$

where $\bar{\sigma}$, $\bar{\zeta}$, χ, and $\bar{s}^M$ are defined as before.

In Section 2, we used Shephard's Lemma to characterize $\frac{\mathrm{d}\ln z_h/z}{\mathrm{d}\ln w/r}$. Here, we do not presume that K_h, L_h, and M_h minimize the plant's static cost, so we cannot make use of the envelope theorem. Instead, differentiating z_h with respect to relative factor prices and rearranging yields:

$$\sigma^{agg} = (1-\chi)\bar{\sigma} + \chi \bar{s}^M \bar{\zeta} + \int \frac{(\alpha_h - \alpha)\,\theta_h}{\alpha\,(1-\alpha)} \left[s_h^K \frac{d\ln K_h}{d\ln w/r} + s_h^L \frac{d\ln L_h}{d\ln w/r} + s_h^M \frac{d\ln M_h}{d\ln w/r} \right] dH(h). \quad \text{(I.1)}$$

where s_h^K, s_h^L, and s_h^M are plant h's respective cost shares of capital, labor and materials.

Our long-run cross-sectional estimates of the elasticity of substitution yield estimates of $\bar{\sigma}$ and $\bar{\zeta}$; those estimates capture how plants adjust their relative factor expenditures in response to changes in factor prices, however these expenditures are chosen. As a result, the difference between our baseline estimate $\hat{\sigma}^{agg}$ and the true underlying aggregate elasticity is:

$$\hat{\sigma}^{agg} - \sigma^{agg} = (1-\bar{s}^M)\chi\varepsilon - \int \frac{(\alpha_h - \alpha)\,\theta_h}{\alpha(1-\alpha)} \left[s_h^K \frac{\mathrm{d}\ln K_h}{\mathrm{d}\ln w/r} + s_h^L \frac{\mathrm{d}\ln L_h}{\mathrm{d}\ln w/r} + s_h^M \frac{\mathrm{d}\ln M_h}{\mathrm{d}\ln w/r} \right] dH(h). \quad \text{(I.2)}$$

Exogenous Wedges We first study an environment motivated by Hsieh and Klenow (2009) and Restuccia and Rogerson (2008) in which plants behave as if there are plant-specific taxes on each input. While plant i's cost of capital, labor, and intermediate inputs are r, w, and q, respectively, it behaves as if these costs were $(1+\tau_{Ki})r$, $(1+\tau_{Li})w$, and $(1+\tau_{Mi})q$. We assume that the distortions themselves do not change with relative factor prices. In that case, the aggregate elasticity of substitution σ^{agg} is our baseline estimate $\hat{\sigma}^{agg}$ plus a distortion term:

$$\sigma^{agg} = \hat{\sigma}^{agg} + \sum_i \frac{(\alpha_i - \alpha)\,\theta_i}{\alpha(1-\alpha)} \left[X_i^1 (\varepsilon - \sigma_i) + X_i^2 (\varepsilon - \zeta_i) \right],$$

where

$$X_i^1 \equiv (1-s_i^M)\alpha_i(1-\alpha_i) \frac{\tau_{Ki} - \tau_{Li}}{(1+\tau_{Ki})\left(1-s_i^M\right)\alpha_i + (1+\tau_{Li})\left(1-s_i^M\right)(1-\alpha_i) + (1+\tau_{Mi})s_i^M}$$

$$X_i^2 = (\alpha_i - \alpha)s_i^M(1-s_i^M) \frac{\alpha_i\tau_{Ki} + (1-\alpha_i)\tau_{Li} - \tau_{Mi}}{(1+\tau_{Ki})(1-s_i^M)\alpha_i + (1+\tau_{Li})(1-s_i^M)(1-\alpha_i) + (1+\tau_{Mi})s_i^M}.$$

We use a perturbation approach to further characterize how misallocation affects the aggregate elasticity. The perturbation parameter ν refers to an economy in which the wedges are $1+\nu\tau_{Ki}$, $1+\nu\tau_{Li}$, and $1+\nu\tau_{Mi}$. Thus, a frictionless economy corresponds to $\nu = 0$, while the economy with misallocation corresponds to $\nu = 1$. Taking a first order approximation around $\nu = 0$, the difference between the our baseline estimate of the aggregate elasticity for the manufacturing sector and the true underlying elasticity is

$$\sigma^{agg} - \hat{\sigma}^{agg} \approx \sum_i \frac{(\alpha_i^* - \alpha^*)\theta_i^*}{\alpha^*(1-\alpha^*)} \left\{ \begin{array}{c} \alpha_i^*(1-\alpha_i^*)(\tau_{Ki} - \tau_{Li})(\varepsilon - \sigma_i^*) \\ + \left[\alpha_i^*\tau_{Ki} + (1-\alpha_i^*)\tau_{Li} - \tau_{Mi}\right] s_i^{M*}(1-s_i^{M*})(\alpha_i^* - \alpha^*)(\varepsilon - \zeta_i^*) \end{array} \right\},$$

where variables with stars are the values in the undistorted economy with $\nu = 0$.

If all dispersion in factor shares were due to wedges rather than to changes in technology, so $\alpha_i^* = \alpha^*$, to a first order approximation, our baseline estimate would recover the true aggregate elasticity. In order to proceed beyond this special case, we must model specific mechanisms through which endogenous wedges would covary with plants' technologies and with factor prices. We therefore turn to explicit models of adjustment costs.

Adjustment Costs In this section we study a class of capital adjustment frictions that nests time-dependent frictions such as Calvo (1983) and Taylor (1980) as well as time-to-build adjustment frictions. Formally, we parameterize capital adjustment frictions by $\left\{\bar{\Gamma}_j\right\}_{j=0}^{\infty}$. If a plant is able to choose capital freely in period t, $\bar{\Gamma}_j$ is the probability that that choice determines capital in period

$t+j$.[16] The fraction of plants in the cross-section whose usage of capital was determined by a choice made j periods ago is $\Gamma_j \equiv \frac{\bar{\Gamma}_j}{\sum_{j'=0}^{\infty}\bar{\Gamma}_{j'}}$. A plant with technology τ can produce with the production function $F(\cdot;\tau)$. We assume that τ follows a Markov process with a stationary distribution $T(\tau)$. We also assume initial conditions are such that the each plant's time since last adjustment, j, is independent of its technology, τ.

To make progress, we make two simplifications: First, we assume that plants do not discount the future, which we believe is a reasonable approximation given the horizon of adjustment frictions. Second, we assume that plants with technology τ produce with the CES production function $F(K,L;\tau) \equiv \left[(A_\tau K)^{\frac{\sigma-1}{\sigma}} + (B_\tau L)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$.

Plants may choose capital only occasionally, so a decision today may determine future input usage. Without adjustment frictions, a plant would tailor its inputs to match its technology and demand period by period. With adjustment frictions, a plant can only match its capital to its shocks in expectation. This difference will affect how plants' scales change with a change in relative factor prices.

Thus, a plant's choice of capital satisfies $\mathbb{E}_t\left[\sum_{j=0}^{\infty}\Gamma_j(MRPK_{i,t+j}-r)\right]$, where $MRPK_{i,t+j}$ is i's marginal revenue product of capital, whereas its choice of labor is a static decision that satisfies $MRPL_{i,t+j}=w$. To find how plants' input usage changes with relative factor prices, we can simply differentiate these equation with respect to relative factor prices.

In this context, we show in Web Appendix I.3 that the difference between our baseline estimate and the true elasticity of substitution can be expressed as

$$\sigma^{agg} - \hat{\sigma}^{agg} = \int x(\tau)\frac{rK(\tau)}{rK+wL}dT(\tau) \tag{I.3}$$

where $T(\tau)$ is the cross-sectional distribution of technology states and $x(\tau)$ is, for a plant that has technology τ when it chooses its capital, the probability-weighted covariance of its realized capital share $\alpha(\tau_{t+j};\tau_t)$ with the sensitivity of scale in that state to long-run factor prices $b(\tau_{t+j};\tau_t)$ over the horizon over which that choice is in effect:[17]

$$x(\tau) \equiv \mathbb{E}_t\left[\sum_{j=0}^{\infty}\Gamma_j\,(\alpha(\tau_{t+j};\tau_t)-\bar{\alpha}(\tau_t))\left(\frac{b(\tau_{t+j};\tau_t)}{\bar{b}(\tau_t)}-1\right)\middle|\,\tau_t=\tau\right] \tag{I.4}$$

Compared to our baseline model, the model with adjustment costs makes different inferences about how plants' scale changes when factor prices change, given the same data. In the baseline model, Shephard's Lemma implies that the change in a plant's scale when the cost of capital falls is

[16]For Calvo-style adjustment frictions for which ν is the probability that a plant is able to adjust each period, $\bar{\Gamma}_j=(1-\nu)^j$. For Taylor-style adjustment frictions in which capital can be adjusted every j^* periods, $\bar{\Gamma}_j = 1\{j<j^*\}$. For time-to-build adjustment frictions in which capital chosen today will not be operational j^* periods later, $\bar{\Gamma}_j = 1\{j=j^*\}$

[17]The realized capital share is $\alpha(\tau_{t+j};\tau_t) \equiv \frac{rK(\tau_t)}{rK(\tau_t)+wL(\tau_{t+j},K(\tau_t))}$. Web Appendix I.3 shows that the sensitivity of the scale to long-run factor prices $b(\tau_{t+j};\tau_t) \equiv \frac{\left(\frac{\varepsilon}{\sigma}-1\right)\frac{\tilde{\alpha}(\tau_{t+j},\tau_t)}{\alpha(\tau_{t+j},\tau_t)}+1}{\left(\frac{\varepsilon}{\sigma}-1\right)\tilde{\alpha}(\tau_{t+j},\tau_t)+1}$, where $\tilde{\alpha}(\tau_{t+j},\tau_t) \equiv \frac{\left(A_{\tau_{t+j}}K(\tau_t)\right)^{\frac{\sigma-1}{\sigma}}}{\left(A_{\tau_{t+j}}K(\tau_t)\right)^{\frac{\sigma-1}{\sigma}}+\left(B_{\tau_{t+j}}L(\tau_{t+j},K(\tau_t))\right)^{\frac{\sigma-1}{\sigma}}}$ is the shadow capital share of the state. $\bar{\alpha}(\tau) = \sum_{j=0}^{\infty}\Gamma_j\mathbb{E}_t\left[\alpha(\tau_{t+j};\tau_t)\middle|\,\tau_t=\tau\right]$ and $\bar{b}(\tau) \equiv \sum_{j=0}^{\infty}\Gamma_j\mathbb{E}_t\left[b(\tau_{t+j};\tau_t)\middle|\,\tau_t=\tau\right]$ are the probability-weighted averages of α and b over the spell of non-adjustment.

proportional to its actual capital share. Thus variation in capital shares in the cross section reflects scope for reallocation across plants in response to changes in factor prices.

With adjustment costs, the variation in capital shares in the cross-section can be decomposed into variation within non-adjustment spells and variation across non-adjustment spells. Like in the baseline model, the variation across spells measures the scope for reallocation: a permanent increase in the relative cost of labor causes plants that expect to be more capital intensive to choose capital at the beginning of the spell and to choose labor during the spell so that they expand more, on average, than plants that expect to be labor intensive. In contrast, a plant is limited in how it can reallocate resources from capital-intensive states to labor-intensive states within a spell.

If shocks tend to be Hicks-neutral, we can show analytically that $x(\tau) < 0$ for all τ. That is, given the same data, the model with adjustment frictions infers less scope than the baseline model for reallocation between capital-intensive and labor intensive plants/states. If shocks are non-neutral, it is possible that $x(\tau)$ can be positive for some τ. For example, if shocks tend to be purely capital augmenting, plants that expect to be extremely labor intensive ($\bar{\alpha}(\tau) < 0.15$) will make choices of labor that are sufficiently different across states that the model with adjustment costs infers more scope for reallocation than the baseline model. We cannot measure $b(\tau_{t+j}, \tau_t)$ directly because we cannot observe the bias of technology. Therefore, to assess the sign of the overall bias, we examine four scenarios: shocks are purely Hicks-neutral; shocks are purely labor-augmenting; shocks are purely capital-augmenting; shocks to A and B are perfectly negatively correlated. For each we take a second-order approximation of each $x(\tau)$ around a fixed-technology benchmark, and approximate the distribution of $\bar{\alpha}(\tau)$ using the empirical distribution of capital shares. As we detail in Appendix I.3, the actual aggregate elasticity would be lower than our baseline estimate in all four cases.

The magnitude of the bias is increasing in the within-spell variation in technology. To gauge the magnitude, we posit that cross-sectional distribution of technology was generated by an autoregressive process. As an upper bound, we study case in which technology has no persistence—the IID case—which maximizes within-spell variation in technology. In that case, the difference between our baseline estimate and the true aggregate elasticity is 0.026.

Plant-Specific Prices Lastly, we consider an environment in which plants pay idiosyncratic prices for their inputs. Formally, plant i pays factor prices $r_i = (1 + \tau_{Ki})r$, $w_i = (1 + \tau_{Li})w$, and $q_i = (1 + \tau_{Mi})q$, where the plant-specific input-price premium might reflect compensating differentials or supplier markups. For example, our identification of the plant-level elasticity of substitution relies on plants in different locations facing different wages. We define the aggregate elasticity of substitution as how factor shares respond to relative factor prices[18]:

$$\sigma^{agg} - 1 = \frac{\mathrm{d}\ln \frac{\alpha}{1-\alpha}}{\mathrm{d}\ln w/r}$$

where $\alpha \equiv \frac{\sum_i r_i K_i}{\sum_i r_i K_i + w_i L_i}$ and the derivative is taken holding fixed the input price premia, $\{\tau_{Ki}, \tau_{Li}, \tau_{Mi}\}$.

In this context, the aggregate elasticity of substitution is exactly the same as our baseline expression in Proposition 1, provided that we define the shares in terms of factor payments that include plant-specific prices. Thus, as long as expenditures are measured correctly, no modifications are necessary.

[18] In this environment, changes in the capital-labor ratio do not map directly into changes in factor compensation, so $\frac{\mathrm{d}\ln K/L}{\mathrm{d}\ln w/r} - 1 \neq \frac{\mathrm{d}\ln \frac{\alpha}{1-\alpha}}{\mathrm{d}\ln w/r}$.

I.2 Proofs for General Framework

We begin by deriving an expression for the aggregate elasticity. We identify with each plant a history of shocks, h. The plant's usage of capital, labor, and materials, K_h, L_h, and M_h, are measurable with respect to this history, and may or may not reflect choices made in the past. Let $H(h)$ be the distribution of histories. These determine the plant's capital share, $\alpha_h \equiv \frac{rK_h}{rK_h+wL_h}$ and share of expenditures, $\theta_h \equiv \frac{rK_h+wL_h}{rK+wL}$. Similarly, define $s_h^K \equiv \frac{rK_h}{rK_h+wL_h+qM_h}$, $s_h^L \equiv \frac{wL_h}{rK_h+wL_h+qM_h}$, and $s_h^M \equiv \frac{qM_h}{rK_h+wL_h+qM_h}$ to be plant h's expenditure on each input as a share of its cost, as well as $\frac{z_h}{z} \equiv \frac{rK_h+wL_h+qM_h}{rK+wL+qM}$ to be its expenditure on inputs as a share of total expenditure.

The aggregate capital share is $\alpha = \int \alpha_h \theta_h dH(h)$. The long-run elasticity of substitution is $\sigma^{agg} - 1 \equiv \frac{\mathrm{d}\ln\frac{\alpha}{1-\alpha}}{\mathrm{d}\ln w/r} = \frac{1}{\alpha(1-\alpha)}\frac{\mathrm{d}\alpha}{\mathrm{d}\ln w/r}$. In addition, define $\sigma_h \equiv \frac{\mathrm{d}\ln\frac{\alpha_h}{1-\alpha_h}}{\mathrm{d}\ln w/r} = \frac{1}{\alpha_h(1-\alpha_h)}\frac{\mathrm{d}\alpha_h}{\mathrm{d}\ln w/r}$ to be the change in plant h's relative factor expenditures in response to a permanent change in factor shares. Finally, define ζ_h to satisfy $(\alpha^M - \alpha_h)(\zeta_h - 1) = \frac{\mathrm{d}\ln\frac{s_h^M}{1-s_h^M}}{\mathrm{d}\ln w/r} = -\frac{1}{s_h^M}\frac{\mathrm{d}\ln 1-s_h^M}{\mathrm{d}\ln w/r}$. Note that σ_h and ζ_h are defined as local behavioral elasticities, which may or may not correspond to the the curvature locally and may reflect both the curvature of the production function and the history h. Following exactly the steps of Section 2.1, the definitions of σ^{agg} and σ_h along with $\frac{\mathrm{d}\alpha}{\mathrm{d}\ln w/r} = \int \frac{\mathrm{d}[\alpha_h\theta_h]}{\mathrm{d}\ln w/r}dH(h)$ gives

$$\begin{aligned}
\alpha(1-\alpha)(\sigma^{agg}-1) &= \frac{\mathrm{d}\alpha}{\mathrm{d}\ln w/r} = \int \left[\frac{\mathrm{d}\alpha_h}{\mathrm{d}\ln w/r}\theta_h + \alpha_h\frac{\mathrm{d}\theta_h}{\mathrm{d}\ln w/r}\right]dH(h) \\
&= \int \alpha_h(1-\alpha_h)\frac{\mathrm{d}\ln\frac{\alpha_h}{1-\alpha_h}}{\mathrm{d}\ln w/r}\theta_h dH(h) + \int (\alpha_h-\alpha)\theta_h\frac{\mathrm{d}\ln\theta_h}{\mathrm{d}\ln w/r}dH(h) \\
&= \int \alpha_h(1-\alpha_h)(\sigma_h-1)\theta_h dH(h) + \int (\alpha_h-\alpha)\theta_h\frac{\mathrm{d}\ln\theta_h}{\mathrm{d}\ln w/r}dH(h)
\end{aligned}$$

We also have $\theta_h = \frac{1-s_h^M}{1-s^M}\frac{z_h}{z}$. This along with the definition of ζ_h implies

$$\begin{aligned}
\int (\alpha_h-\alpha)\theta_h\frac{\mathrm{d}\ln\theta_h}{\mathrm{d}\ln w/r}dH(h) &= \int (\alpha_h-\alpha)\theta_h\left(\frac{\mathrm{d}\ln(1-s_h^M)}{\mathrm{d}\ln w/r} + \frac{\mathrm{d}\ln z_h/z}{\mathrm{d}\ln w/r}\right)dH(h) \\
&= \int (\alpha_h-\alpha)\theta_h\left((\alpha_h-\alpha^M)s_h^M(\zeta_h-1) + \frac{\mathrm{d}\ln z_h/z}{\mathrm{d}\ln w/r}\right)dH(h)
\end{aligned}$$

Dividing by $\alpha(1-\alpha)$ and using the definitions of χ, $\bar{\sigma}$, $\bar{\zeta}$, and $\bar{s}^M$, we have

$$\sigma^{agg} - 1 = (1-\chi)(\bar{\sigma}-1) + \chi\bar{s}^M(\bar{\zeta}-1) + \int_h \frac{(\alpha_h-\alpha)\theta_h}{\alpha(1-\alpha)}\frac{\mathrm{d}\ln z_h/z}{\mathrm{d}\ln w/r}dH(h)$$

We next find a simpler way to express the final term. We first note that

$$\begin{aligned}
\frac{d\ln z_h/z}{d\ln w/r} &= \frac{d\ln K_h + \frac{w}{r}L_h + \frac{q}{r}M_h}{d\ln w/r} - \frac{d\ln K + \frac{w}{r}L + \frac{q}{r}M}{d\ln w/r} \\
&= s_h^K\frac{d\ln K_h}{d\ln w/r} + s_h^L\left(1 + \frac{d\ln L_h}{d\ln w/r}\right) + s_h^M\left(1 - \alpha^M + \frac{d\ln M_h}{d\ln w/r}\right) - \frac{d\ln K + \frac{w}{r}L + \frac{q}{r}M}{d\ln w/r}
\end{aligned}$$

Plugging this into the integral yields

$$\int \frac{(\alpha_h - \alpha)\theta_h}{\alpha(1-\alpha)} \frac{d\ln z_h/z}{d\ln w/r} dH(h) = \int \frac{(\alpha_h - \alpha)\theta_h}{\alpha(1-\alpha)} \begin{bmatrix} s_h^K \frac{d\ln K_h}{d\ln w/r} + s_h^L \left(1 + \frac{d\ln L_h}{d\ln w/r}\right) \\ + s_h^M \left(1 - \alpha^M + \frac{d\ln M_h}{d\ln w/r}\right) \end{bmatrix} dH(h)$$

Next, note that

$$\begin{aligned}
&\int \frac{(\alpha_h - \alpha)\theta_h}{\alpha(1-\alpha)} \left[s_h^L + s_h^M(1-\alpha^M)\right] dH(h) \\
&= \int \frac{(\alpha_h - \alpha)\theta_h}{\alpha(1-\alpha)} \left[(1-s_h^M)(1-\alpha_h) + s_h^M(1-\alpha^M)\right] dH(h) \\
&= \int \frac{(\alpha_h - \alpha)\theta_h}{\alpha(1-\alpha)} (1-\alpha_h) dH(h) + \int \frac{(\alpha_h - \alpha)\theta_h}{\alpha(1-\alpha)} \left[\alpha_h - \alpha^M\right] s_h^M dH(h) \\
&= -\chi + \bar{s}^M \chi
\end{aligned}$$

Together, these imply

$$\sigma^{agg} = (1-\chi)\bar{\sigma} + \chi(1-\bar{s}^M)\bar{\zeta} + \int \frac{(\alpha_h - \alpha)\,\theta_h}{\alpha\,(1-\alpha)} \left[s_h^K \frac{d\ln K_h}{d\ln w/r} + s_h^L \frac{d\ln L_h}{d\ln w/r} + s_h^M \frac{d\ln M_h}{d\ln w/r}\right] dH(h)$$

I.3 Proofs for Implicit Taxes

The aggregate capital share can be expressed as a weighted average of the individual capital shares

$$\alpha = \sum_i \alpha_i \theta_i = \sum_i \alpha_i \frac{1-s_i^M}{1-s^M} \frac{z_i}{z} \tag{I.5}$$

where $\alpha_i = \frac{rK_i}{rK_i + wL_i}$, $s_i^M = \frac{qM_i}{rK_i + wL_i + qM_i}$, and $\frac{z_i}{z} = \frac{rK_i + wL_i + qM_i}{rK + wL + qM}$ are defined in terms of plant i's actual expenditures. We find the aggregate elasticity by differentiating with respect to relative factor prices.

Claim I.1 *In an environment with misallocation, the aggregate elasticity of substitution between capital and labor is*

$$\sigma^{agg} = \hat{\sigma}^{agg} + \sum_i \frac{(\alpha_i - \alpha)\,\theta_i}{\alpha(1-\alpha)} \left[X_i^1(\varepsilon - \sigma_i) + X_i^2(\varepsilon - \zeta_i)\right]$$

where

$$\begin{aligned}
\hat{\sigma}^{agg} &\equiv \chi\bar{\sigma} + (1-\chi)\left[\bar{s}^M\bar{\zeta} + \left(1-\bar{s}^M\right)\varepsilon\right] \\
X_i^1 &\equiv (1-s_i^M)\alpha_i(1-\alpha_i)\frac{\tau_{Ki} - \tau_{Li}}{(1+\tau_{Ki})\left(1-s_i^M\right)\alpha_i + (1+\tau_{Li})\left(1-s_i^M\right)(1-\alpha_i) + (1+\tau_{Mi})s_i^M} \\
X_i^2 &= (\alpha_i - \alpha)s_i^M(1-s_i^M)\frac{\alpha_i\tau_{Ki} + (1-\alpha_i)\tau_{Li} - \tau_{Mi}}{(1+\tau_{Ki})(1-s_i^M)\alpha_i + (1+\tau_{Li})(1-s_i^M)(1-\alpha_i) + (1+\tau_{Mi})s_i^M}
\end{aligned}$$

Proof. Differentiating (I.5) with respect to relative factor prices gives

$$\frac{\mathrm{d}\alpha}{\mathrm{d}\ln w/r} = \sum_i \frac{\mathrm{d}\alpha_i}{\mathrm{d}\ln w/r}\theta_i + \sum_i (\alpha_i - \alpha)\,\theta_i \frac{\mathrm{d}\ln(1-s_i^M)}{\mathrm{d}\ln w/r} + \sum_i (\alpha_i - \alpha)\,\theta_i \frac{\mathrm{d}\ln z_i/z}{\mathrm{d}\ln w/r}$$

The aggregate elasticity of substitution satisfies $\sigma^{agg} - 1 = \frac{1}{\alpha(1-\alpha)}\frac{\mathrm{d}\alpha}{\mathrm{d}\ln w/r}$. The plant-level elasticity of substitution between capital and labor satisfies $\sigma_i = \frac{\mathrm{d}\ln K_i/L_i}{\mathrm{d}\ln w/r}$, so that, as in the baseline, $\sigma_i - 1 = \frac{\mathrm{d}\ln\frac{\alpha_i}{1-\alpha_i}}{\mathrm{d}\ln w/r} = \frac{1}{\alpha_i(1-\alpha_i)}\frac{\mathrm{d}\alpha_i}{\mathrm{d}\ln w/r}$. Similarly, we define the plant-level elasticity of substitution between primary inputs and intermediates to satisfy $(\alpha_i - \alpha^M)(1-\zeta_i) = \frac{\mathrm{d}\ln\frac{s_i^M}{1-s_i^M}}{\mathrm{d}\ln w/r}$, so that $\frac{\mathrm{d}\ln(1-s_i^M)}{\mathrm{d}\ln w/r} = s_i^M(\alpha_i - \alpha^M)(\zeta_i - 1)$. Using these expressions, the aggregate elasticity of substitution can be expressed as

$$\begin{aligned}\alpha(1-\alpha)(\sigma^{agg}-1) &= \sum_i \alpha_i(1-\alpha_i)(\sigma_i - 1)\theta_i + \sum_i (\alpha_i - \alpha)\theta_i s_i^M(\alpha_i - \alpha^M)(\zeta_i - 1)\\ &\quad + \sum_i \frac{(\alpha_i-\alpha)\theta_i}{\alpha(1-\alpha)}\frac{\mathrm{d}\ln z_i/z}{\mathrm{d}\ln w/r}\end{aligned}$$

Dividing through by $\alpha(1-\alpha)$ and using the expression for $\bar{\sigma}$, $\bar{s}^M$, and $\bar{\zeta}$, gives

$$\sigma^{agg} - 1 = (1-\chi)(\bar{\sigma} - 1) + \chi \bar{s}^M\left(\bar{\zeta} - 1\right) + \sum_i \frac{(\alpha_i - \alpha)\,\theta_i}{\alpha(1-\alpha)}\frac{\mathrm{d}\ln z_i/z}{\mathrm{d}\ln w/r} \tag{I.6}$$

We now derive an expression for $\frac{\mathrm{d}\ln z_i/z}{\mathrm{d}\ln w/r}$. Note that

$$\frac{z_i}{z} = \frac{rK_i + wL_i + qM_i}{rK + wL + qM} = \frac{\frac{\varepsilon-1}{\varepsilon}PY}{rK + wL + qM}\frac{rK_i + wL_i + qM_i}{\frac{\varepsilon-1}{\varepsilon}P_iY_i}\frac{P_iY_i}{PY}$$

Using $\frac{P_iY_i}{PY} = (P_i/P)^{1-\varepsilon} = (P_i/r)^{1-\varepsilon}(r/P)^{1-\varepsilon}$, we have

$$\frac{z_i}{z} = \frac{\frac{\varepsilon-1}{\varepsilon}PY\,(r/P)^{1-\varepsilon}}{rK + wL + qM}\frac{rK_i + wL_i + qM_i}{\frac{\varepsilon-1}{\varepsilon}P_iY_i}(P_i/r)^{1-\varepsilon}$$

Then using the fact that for any constant ψ, $\sum_i \frac{(\alpha_i-\alpha)\theta_i}{\alpha(1-\alpha)}\psi = 0$, we have

$$\sum_i \frac{(\alpha_i - \alpha)\,\theta_i}{\alpha\,(1-\alpha)}\frac{\mathrm{d}\ln z_i/z}{\mathrm{d}\ln w/r} = (1-\varepsilon)\sum_i \frac{(\alpha_i - \alpha)\,\theta_i}{\alpha\,(1-\alpha)}\frac{\mathrm{d}\ln P_i/r}{\mathrm{d}\ln w/r} - \sum_i \frac{(\alpha_i - \alpha)\,\theta_i}{\alpha(1-\alpha)}\frac{\mathrm{d}\ln\frac{\frac{\varepsilon-1}{\varepsilon}P_iY_i}{rK_i + wL_i + qM_i}}{\mathrm{d}\ln w/r} \tag{I.7}$$

For the first term, let $C_i(r, w, q)$ be the unit cost function associated with i's production technology. Firm i's optimal price is

$$P_i = \frac{\varepsilon}{\varepsilon - 1}C_i\left((1+\tau_{Ki})\,r, (1+\tau_{Li})w, (1+\tau_{Mi})q\right)$$

Shephard's lemma implies that

$$\begin{aligned}
\frac{\mathrm{d}\ln P_i/r}{\mathrm{d}\ln w/r} &= \frac{\mathrm{d}\ln C_i\left((1+\tau_{Ki}),(1+\tau_{Li})\frac{w}{r},(1+\tau_{Mi})\frac{q}{r}\right)}{\mathrm{d}\ln w/r} \\
&= \frac{(1+\tau_{Li})wC_{iw}}{C_i}+\frac{\mathrm{d}\ln q/r}{\mathrm{d}\ln w/r}\frac{(1+\tau_{Mi})qC_{iq}}{C_i} \\
&= \frac{(1+\tau_{Li})wL_i}{(1+\tau_{Ki})rK_i+(1+\tau_{Li})wL_i+(1+\tau_{Mi})qM_i} \\
&\quad +\left(1-\alpha^M\right)\frac{(1+\tau_{Mi})qM_i}{(1+\tau_{Ki})rK_i+(1+\tau_{Li})wL_i+(1+\tau_{Mi})qM_i} \\
&= \frac{(1+\tau_{Li})wL_i+(1-\alpha_i)(1+\tau_{Mi})qM_i}{(1+\tau_{Ki})rK_i+(1+\tau_{Li})wL_i+(1+\tau_{Mi})qM_i} \\
&\quad +\left(\alpha_i-\alpha^M\right)\frac{(1+\tau_{Mi})qM_i}{(1+\tau_{Ki})\,rK_i+(1+\tau_{Li})wL_i+(1+\tau_{Mi})\,qM_i}
\end{aligned}$$

Define X_i^1 and X_i^2 as

$$\begin{aligned}
X_i^1 &\equiv (1-\alpha_i)-\frac{(1+\tau_{Li})\,wL_i+(1-\alpha_i)(1+\tau_{Mi})qM_i}{(1+\tau_{Ki})rK_i+(1+\tau_{Li})wL_i+(1+\tau_{Mi})qM_i} \\
X_i^2 &\equiv (\alpha_i-\alpha^M)s_i^M-\left(\alpha_i-\alpha^M\right)\frac{(1+\tau_{Mi})qM_i}{(1+\tau_{Ki})rK_i+(1+\tau_{Li})wL_i+(1+\tau_{Mi})\,qM_i}
\end{aligned}$$

Then

$$\begin{aligned}
\sum_i \frac{(\alpha_i-\alpha)\,\theta_i}{\alpha\,(1-\alpha)}\frac{\mathrm{d}\ln P_i/r}{\mathrm{d}\ln w/r} &= \sum_i \frac{(\alpha_i-\alpha)\,\theta_i}{\alpha(1-\alpha)}\left[(1-\alpha_i)-X_i^1+\left(\alpha_i-\alpha^M\right)s_i^M-X_i^2\right] \\
&= -\left\{\left(1-\bar{s}^M\right)\chi+\sum_i\frac{(\alpha_i-\alpha)\,\theta_i}{\alpha(1-\alpha)}\left(X_i^1+X_i^2\right)\right\} \qquad \text{(I.8)}
\end{aligned}$$

Note that we can derive more explicit expressions for X_i^1 and X_i^2:

$$\begin{aligned}
X_i^1 &= (1-\alpha_i)-\frac{(1+\tau_{Li})wL_i+(1-\alpha_i)(1+\tau_{Mi})qM_i}{(1+\tau_{Ki})rK_i+(1+\tau_{Li})wL_i+(1+\tau_{Mi})qM_i} \\
&= (1-\alpha_i)-\frac{(1+\tau_{Li})(1-s_i^M)(1-\alpha_i)+(1-\alpha_i)(1+\tau_{Mi})s_i^M}{(1+\tau_{Ki})(1-s_i^M)\alpha_i+(1+\tau_{Li})(1-s_i^M)(1-\alpha_i)+(1+\tau_{Mi})s_i^M} \\
&= (1-\alpha_i)\left\{1-\frac{(1+\tau_{Li})(1-s_i^M)+(1+\tau_{Mi})s_i^M}{(1+\tau_{Ki})(1-s_i^M)\alpha_i+(1+\tau_{Li})(1-s_i^M)(1-\alpha_i)+(1+\tau_{Mi})s_i^M}\right\} \\
&= (1-s_i^M)\alpha_i(1-\alpha_i)\frac{\tau_{Ki}-\tau_{Li}}{(1+\tau_{Ki})(1-s_i^M)\alpha_i+(1+\tau_{Li})(1-s_i^M)(1-\alpha_i)+(1+\tau_{Mi})s_i^M}
\end{aligned}$$

and

$$
\begin{aligned}
X_i^2 &= (\alpha_i-\alpha^M)s_i^M-(\alpha_i-\alpha^M)\frac{(1+\tau_{Mi})qM_i}{(1+\tau_{Ki})rK_i+(1+\tau_{Li})wL_i+(1+\tau_{Mi})qM_i}\\
&= (\alpha_i-\alpha^M)s_i^M-(\alpha_i-\alpha^M)\frac{(1+\tau_{Mi})s_i^M}{(1+\tau_{Ki})(1-s_i^M)\alpha_i+(1+\tau_{Li})(1-s_i^M)(1-\alpha_i)+(1+\tau_{Mi})s_i^M}\\
&= (\alpha_i-\alpha^M)s_i^M\left\{1-\frac{(1+\tau_{Mi})}{(1+\tau_{Ki})(1-s_i^M)\alpha_i+(1+\tau_{Li})(1-s_i^M)(1-\alpha_i)+(1+\tau_{Mi})s_i^M}\right\}\\
&= (\alpha_i-\alpha^M)s_i^M(1-s_i^M)\frac{\alpha_i\tau_{Ki}+(1-\alpha_i)\tau_{Li}-\tau_{Mi}}{(1+\tau_{Ki})(1-s_i^M)\alpha_i+(1+\tau_{Li})(1-s_i^M)(1-\alpha_i)+(1+\tau_{Mi})s_i^M}
\end{aligned}
$$

For the second term, we have

$$
\begin{aligned}
\frac{\mathrm{d}\ln\frac{\frac{\varepsilon-1}{\varepsilon}P_iY_i}{rK_i+wL_i+qM_i}}{\mathrm{d}\ln w/r} &= \frac{\mathrm{d}\ln\frac{(1+\tau_{Ki})rK_i+(1+\tau_{Li})wL_i+(1+\tau_{Mi})qM_i}{rK_i+wL_i+qM_i}}{\mathrm{d}\ln w/r}\\
&= \frac{\mathrm{d}\ln\left[(1+\tau_{Ki})(1-s_i^M)\alpha_i+(1+\tau_{Li})(1-s_i^M)(1-\alpha_i)+(1+\tau_{Mi})s_i^M\right]}{\mathrm{d}\ln w/r}\\
&= \frac{\left\{\begin{array}{c}\left[(1-s_i^M)(1+\tau_{Ki})-(1-s_i^M)(1+\tau_{Li})\right]\frac{\mathrm{d}\alpha_i}{\mathrm{d}\ln w/r}\\ -\left[(1+\tau_{Ki})\alpha_i+(1-\alpha_i)(1+\tau_{Li})-(1+\tau_{Mi})\right]\frac{ds_i^M}{\mathrm{d}\ln w/r}\end{array}\right\}}{(1+\tau_{Ki})(1-s_i^M)\alpha_i+(1+\tau_{Li})(1-s_i^M)(1-\alpha_i)+(1+\tau_{Mi})s_i^M}\\
&= \frac{(1-s_i^M)(\tau_{Ki}-\tau_{Li})\frac{\mathrm{d}\alpha_i}{\mathrm{d}\ln w/r}-\left[\alpha_i\tau_{Ki}+(1-\alpha_i)\tau_{Li}-\tau_{Mi}\right]\frac{ds_i^M}{\mathrm{d}\ln w/r}}{(1+\tau_{Ki})(1-s_i^M)\alpha_i+(1+\tau_{Li})(1-s_i^M)(1-\alpha_i)+(1+\tau_{Mi})s_i^M}
\end{aligned}
$$

Using $\frac{\mathrm{d}\alpha_i}{\mathrm{d}\ln w/r}=\alpha_i(1-\alpha_i)(\sigma_i-1)$ and $\frac{ds_i^M}{\mathrm{d}\ln w/r}=-s_i^M(1-s_i^M)(\alpha_i-\alpha^M)(\zeta_i-1)$, this can be expressed as

$$
\frac{\mathrm{d}\ln\frac{\frac{\varepsilon-1}{\varepsilon}P_iY_i}{rK_i+wL_i+qM_i}}{\mathrm{d}\ln w/r}=X_i^1(\sigma_i-1)+X_i^2(\zeta_i-1) \tag{I.9}
$$

Plugging (I.8) and (I.9) into (I.7) gives

$$
\sum_i\frac{(\alpha_i-\alpha)\theta_i}{\alpha(1-\alpha)}\frac{\mathrm{d}\ln z_i/z}{\mathrm{d}\ln w/r}=(1-\bar{s}^M)\chi(\varepsilon-1)+\sum_i\frac{(\alpha_i-\alpha)\theta_i}{\alpha(1-\alpha)}\left[X_i^1(\varepsilon-\sigma_i)+X_i^2(\varepsilon-\zeta_i)\right]
$$

This along with (I.6) gives

$$
\sigma^{agg}=\hat{\sigma}^{agg}+\sum_i\frac{(\alpha_i-\alpha)\theta_i}{\alpha(1-\alpha)}\left[X_i^1(\varepsilon-\sigma_i)+X_i^2(\varepsilon-\zeta_i)\right]
$$

■

We now use a perturbation approach to analyze this expression. Consider an economy indexed by ν for which the wedges are $(1+\nu\tau_{Ki})$, $(1+\nu\tau_{Li})$, and $(1+\nu\tau_{Mi})$. The baseline economy corresponds to $\nu=0$ while the economy with misallocation corresponds to $\nu=1$. We take a first order approximation of $\sigma^{agg}-\hat{\sigma}^{agg}$ by taking a Taylor expansion around $\nu=0$. Note that

$X_i^1(0) = X_i^2(0) = 0$, and that

$$\begin{aligned} X_i^{1'}(0) &= \left(1 - s_i^{M*}\right) \alpha_i^* \left(1 - \alpha_i^*\right) \left(\tau_{Ki} - \tau_{Li}\right) \\ X_i^{2'}(0) &= \left(\alpha_i^* - \alpha^*\right) s_i^{M*} \left(1 - s_i^{M*}\right) \left[\alpha_i^* \tau_{Ki} + \left(1 - \alpha_i^*\right) \tau_{Li} - \tau_{Mi}\right] \end{aligned}$$

We therefore have that, to a first order,

$$\begin{aligned} \sigma^{agg}(\nu) - \hat{\sigma}^{agg}(\nu) &\approx \sigma^{agg}(0) - \hat{\sigma}^{agg}(0) + \sigma^{agg'}(0) - \hat{\sigma}^{agg'}(0) \\ &= \sum_i X_i^1(0) \left. \frac{d\left\{\frac{(\alpha_i(\nu)-\alpha(\nu))\theta_i(\nu)}{\alpha(\nu)(1-\alpha(\nu))}\left(\varepsilon(\nu) - \sigma_i(\nu)\right)\right\}}{d\nu} \right|_{\nu=0} \\ &\quad + \sum_i X_i^2(0) \left. \frac{d\frac{(\alpha_i(\nu)-\alpha(\nu))\theta_i(\nu)}{\alpha(\nu)(1-\alpha(\nu))}\left(\varepsilon(\nu) - \zeta_i(\nu)\right)}{d\nu} \right|_{\nu=0} \\ &\quad + \sum_i \frac{\left(\alpha_i^* - \alpha^*\right)\theta_i^*}{\alpha^*\left(1 - \alpha^*\right)} \left[X_i^{1'}(0)\left(\varepsilon - \sigma_i^*\right) + X_i^{2'}(0)\left(\varepsilon - \zeta_i^*\right)\right] \\ &= \sum_i \frac{\left(\alpha_i^* - \alpha^*\right)\theta_i^*}{\alpha^*\left(1 - \alpha^*\right)} \left\{ \begin{array}{c} \left(1 - s_i^{M*}\right)\alpha_i^*\left(1 - \alpha_i^*\right)\left(\tau_{Ki} - \tau_{Li}\right)\left(\varepsilon - \sigma_i^*\right) \\ + \left(\alpha_i^* - \alpha^*\right) s_i^{M*}\left(1 - s_i^{M*}\right)\left[\alpha_i^*\tau_{Ki} + \left(1 - \alpha_i^*\right)\tau_{Li} - \tau_{Mi}\right]\left(\varepsilon - \zeta_i^*\right) \end{array} \right\} \end{aligned}$$

I.4 Proofs for Adjustment Frictions

As discussed in the main text, capital adjustment frictions are parameterized by $\{\bar{\Gamma}_j\}_{j=0}^{\infty}$, where $\bar{\Gamma}_j$ is the probability that a choice of capital made during period t determines capital used in period $t+j$. At the beginning of each period, uncertainty about both a plant's technology and whether it will be able to adjust capital is realized. It then chooses labor, materials, and, if feasible, capital, and then produces.

A plant with technology τ uses the production function $F(K, L; \tau) \equiv \left[(A_\tau K)^{\frac{\sigma-1}{\sigma}} + (B_\tau L)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$. If a plant has technology τ_0 and is able to choose capital, its choice maximizes expected profit over the horizon of non-adjustment

$$\max_{K(\tau_0), \{L(\tau_j;\tau_0)\}_{j=0}^{\infty}} \mathbb{E}\left[\left.\sum_{j=0}^{\infty} \bar{\Gamma}_j \left\{PY^{1/\varepsilon} F(K(\tau_0), L(\tau_j;\tau_0);\tau_j)^{\frac{\varepsilon-1}{\varepsilon}} - rK(\tau_0) - wL(\tau_j;\tau_0)\right\}\right| \tau_0\right]$$

The optimal choices of capital and labor satisfy

$$\begin{aligned} 0 &= \mathbb{E}\left[\left.\sum_{j=0}^{\infty} \bar{\Gamma}_j \left\{PY^{\frac{1}{\tilde{\varepsilon}}} \frac{\tilde{\varepsilon} - 1}{\tilde{\varepsilon}} F\left(K(\tau_0), L(\tau_j;\tau_0);\tau_j\right)^{-\frac{1}{\tilde{\varepsilon}}} F_K\left(K(\tau_0), L(\tau_j;\tau_0);\tau_j\right) - r\right\}\right| \tau_0\right] \\ 0 &= PY^{\frac{1}{\varepsilon}} \frac{\varepsilon - 1}{\varepsilon} F\left(K(\tau_0), L(\tau_j;\tau_0);\tau_j\right)^{-\frac{1}{\varepsilon}} G_L\left(K(\tau_0), L\left(\tau_j;\tau_0\right);\tau_j\right) - w \end{aligned}$$

Suppressing the arguments when not necessary for clarity, dividing the FOC for capital by $\sum_{j'=0}^{\infty} \bar{\Gamma}_{j'}$, and using $\Gamma_j \equiv \frac{\bar{\Gamma}_j}{\sum_{j'=0}^{\infty}\Gamma_{j'}}$, these are

$$0 = \sum_{j=0}^{\infty}\Gamma_j \mathbb{E}\left[PY^{\frac{1}{\varepsilon}}\frac{\varepsilon-1}{\varepsilon}F_i^{-\frac{1}{\varepsilon}}F_{iK} - r\middle|\tau_0\right]$$
$$0 = PY^{\frac{1}{\varepsilon}}\frac{\varepsilon-1}{\varepsilon}F_i^{-\frac{1}{\varepsilon}}F_{iL} - w$$

To characterize how choices of capital and labor change with permanent changes in factor prices, we can simply differentiate these first order conditions. As we show in the lemma below, these depend on the plant's shadow capital share, $\tilde{\alpha}_i \equiv \frac{K_i F_{iK}}{F_i}$.[19] The following lemma summarizes the result.

Lemma I.1 *In response to a permanent change in factor prices, the change in scale for a plant whose capital was chosen with technology τ_0 and now has technology τ_j is*

$$\alpha(\tau_j;\tau_0)\frac{\mathrm{d}\ln K(\tau_0)}{\mathrm{d}\ln w/r} + (1-\alpha(\tau_j;\tau_0))\frac{\mathrm{d}\ln L(\tau_j;\tau_0)}{\mathrm{d}\ln w/r} = \frac{b(\tau_j;\tau_0)}{\bar{b}(\tau_0)}\alpha(\tau_j;\tau_0)\varepsilon - \alpha\varepsilon$$

where $b(\tau_j;\tau_0) \equiv \frac{\left(\frac{\varepsilon}{\sigma}-1\right)\frac{\tilde{\alpha}(\tau_j;\tau_0)}{\alpha(\tau_j;\tau_0)}+1}{\left(\frac{\varepsilon}{\sigma}-1\right)\tilde{\alpha}(\tau_j;\tau_0)+1}$ *and* $\bar{b}(\tau_0) \equiv \mathbb{E}\left[\sum_{j=0}^{\infty}\Gamma_j b(\tau_j;\tau_0)\middle|\tau_0\right]$.

Proof. We begin with a preliminary calculation. Using the fact that F_i is homogeneous of degree one, we have

$$\begin{aligned}
\mathrm{d}\ln\left[(F_i/K_i)^{-\frac{1}{\varepsilon}}F_{iK}\right] &\equiv \mathrm{d}\ln\left[\left(\frac{F_i(K_i,L_i)}{K_i}\right)^{-\frac{1}{\varepsilon}}F_{iK}(K_i,L_i)\right] \\
&= \mathrm{d}\ln\left[F_i(1,L_i/K_i)^{-\frac{1}{\varepsilon}}F_{iK}(1,L_i/K_i)\right] \\
&= \left[-\frac{1}{\varepsilon}\frac{(L_i/K_i)F_{iL}(1,L_i/K_i)}{F_i(1,L_i/K_i)} + \frac{(L_i/K_i)F_{iKL}(1,L_i/K_i)}{F_{iK}(1,L_i/K_i)}\right]\mathrm{d}\ln L_i/K_i \\
&= (1-\tilde{\alpha}_i)\left[\frac{1}{\varepsilon}-\frac{1}{\sigma}\right]\mathrm{d}\ln K_i/L_i
\end{aligned}$$

Similarly,
$$\mathrm{d}\ln\left[(F_i/L_i)^{-\frac{1}{\varepsilon}}F_{iL}\right] = -\tilde{\alpha}_i\left[\frac{1}{\varepsilon}-\frac{1}{\sigma}\right]\mathrm{d}\ln K_i/L_i$$

Multiplying the capital FOC by $\frac{K_i^{1/\varepsilon}}{PY^{1/\varepsilon}}$ and the labor FOC by $\frac{L_i^{1/\varepsilon}}{PY^{1/\varepsilon}}$ gives

$$0 = \mathbb{E}\left[\sum_{j=0}^{\infty}\Gamma_j\frac{\varepsilon-1}{\varepsilon}(F_i/K_i)^{-\frac{1}{\varepsilon}}F_{iK} - \frac{rK_i^{1/\varepsilon}}{PY^{1/\varepsilon}}\right]$$
$$0 = \frac{\varepsilon-1}{\varepsilon}(F_i/L_i)^{-\frac{1}{\varepsilon}}F_{iL} - \frac{wL_i^{1/\varepsilon}}{PY^{1/\varepsilon}}$$

[19] More formally, $\tilde{\alpha}(\tau_j;\tau_0) \equiv \frac{K(\tau_0)F_K(K(\tau_0),L(\tau_j;\tau_0);\tau_j)}{F(K(\tau_0),L(\tau_j;\tau_0);\tau_j)}$.

Differentiating the labor FOC with respect to factor prices gives

$$-\tilde{\alpha}_i\left[\frac{1}{\varepsilon}-\frac{1}{\sigma}\right]\frac{\mathrm{d}\ln K_i/L_i}{\mathrm{d}\ln w/r}=\frac{1}{\varepsilon}\frac{\mathrm{d}\ln L_i/Y}{\mathrm{d}\ln w/r}+\frac{\mathrm{d}\ln w/P}{\mathrm{d}\ln w/r}$$

Using $\frac{\mathrm{d}\ln w/P}{\mathrm{d}\ln w/r}=\alpha$, we can rearrange this as

$$\frac{\mathrm{d}\ln K_i/L_i}{\mathrm{d}\ln w/r}=\frac{\frac{1}{\varepsilon}\frac{\mathrm{d}\ln K_i/Y}{\mathrm{d}\ln w/r}+\alpha}{(1-\tilde{\alpha}_i)\frac{1}{\varepsilon}+\tilde{\alpha}_i\frac{1}{\sigma}} \tag{I.10}$$

Similarly, differentiating the capital FOC with respect to relative factor prices, multiplying through by $-\frac{PY^{1/\varepsilon}}{rK_i^{1/\varepsilon}}$, and using $\frac{\mathrm{d}\ln r/P}{\mathrm{d}\ln w/r}=-(1-\alpha)$ gives

$$\begin{aligned}0 &= \sum_{j=0}^{\infty}\Gamma_j\mathbb{E}\left[\frac{\varepsilon-1}{\varepsilon}\left(F_i/K_i\right)^{-\frac{1}{\varepsilon}}F_{iK}(1-\tilde{\alpha}_i)\left(\frac{1}{\varepsilon}-\frac{1}{\sigma}\right)\frac{\mathrm{d}\ln K_i/L_i}{\mathrm{d}\ln w/r}-\frac{rK_i^{1/\varepsilon}}{PY^{1/\varepsilon}}\left(\frac{1}{\varepsilon}\frac{\mathrm{d}\ln K_i/Y}{\mathrm{d}\ln w/r}+\frac{\mathrm{d}\ln r/P}{\mathrm{d}\ln w/r}\right)\right]\\ &= \sum_{j=0}^{\infty}\Gamma_j\mathbb{E}\left[\frac{PY^{1/\varepsilon}\frac{\varepsilon-1}{\varepsilon}F_i^{\frac{\varepsilon-1}{\varepsilon}}}{rK_i}\frac{K_iF_{iK}}{F_i}(1-\tilde{\alpha}_i)\left(\frac{1}{\sigma}-\frac{1}{\varepsilon}\right)\frac{\mathrm{d}\ln K_i/L_i}{\mathrm{d}\ln w/r}+\left(\frac{1}{\varepsilon}\frac{\mathrm{d}\ln K_i/Y}{\mathrm{d}\ln w/r}-(1-\alpha)\right)\right]\end{aligned}$$

Note that the FOC for labor is $wL_i=PY^{1/\varepsilon}\frac{\varepsilon-1}{\varepsilon}F_i^{-\frac{1}{\varepsilon}}F_{iL}L_i=PY^{1/\varepsilon}\frac{\varepsilon-1}{\varepsilon}F_i^{\frac{\varepsilon-1}{\varepsilon}}(1-\tilde{\alpha}_i)$. Substituting this in gives

$$0=\sum_{j=0}^{\infty}\Gamma_j\mathbb{E}\left[\frac{\tilde{\alpha}_i\,(1-\alpha_i)}{\alpha_i}\left(\frac{1}{\sigma}-\frac{1}{\varepsilon}\right)\frac{\mathrm{d}\ln K_i/L_i}{\mathrm{d}\ln w/r}+\frac{1}{\varepsilon}\frac{\mathrm{d}\ln K_i/Y}{\mathrm{d}\ln w/r}-(1-\alpha)\right]$$

Plugging in the expression for $\frac{\mathrm{d}\ln K_i/L_i}{\mathrm{d}\ln w/r}$ from above gives

$$0 = \sum_{j=0}^{\infty}\Gamma_j\mathbb{E}\left[\frac{\tilde{\alpha}_i(1-\alpha_i)}{\alpha_i}\left(\frac{1}{\sigma}-\frac{1}{\varepsilon}\right)\frac{\frac{1}{\varepsilon}\frac{\mathrm{d}\ln K_i/Y}{\mathrm{d}\ln w/r}+\alpha}{(1-\tilde{\alpha}_i)\frac{1}{\varepsilon}+\tilde{\alpha}_i\frac{1}{\sigma}}+\left(\frac{1}{\varepsilon}\frac{\mathrm{d}\ln K_i/Y}{\mathrm{d}\ln w/r}-(1-\alpha)\right)\right]$$

Using $\sum_{j=0}^{\infty}\Gamma_j=1$, we can rearrange this as

$$\begin{aligned}1 &= \sum_{j=0}^{\infty}\Gamma_j\mathbb{E}\left[\left(\frac{\frac{\tilde{\alpha}_i(1-\alpha_i)}{\alpha_i}\left(\frac{1}{\sigma}-\frac{1}{\varepsilon}\right)}{(1-\tilde{\alpha}_i)\frac{1}{\varepsilon}+\tilde{\alpha}_i\frac{1}{\sigma}}+1\right)\left(\frac{1}{\varepsilon}\frac{\mathrm{d}\ln K_i/Y}{\mathrm{d}\ln w/r}+\alpha\right)\right]\\ &= \sum_{j=0}^{\infty}\Gamma_j\mathbb{E}\left[b_i\left(\frac{1}{\varepsilon}\frac{\mathrm{d}\ln K_i/Y}{\mathrm{d}\ln w/r}+\alpha\right)\right]\\ &= \bar{b}\left(\frac{1}{\varepsilon}\frac{\mathrm{d}\ln K_i/Y}{\mathrm{d}\ln w/r}+\alpha\right)\end{aligned}$$

Using this along with (I.10) gives

$$\begin{aligned}
\alpha_i \frac{\mathrm{d}\ln K_i/Y}{\mathrm{d}\ln w/r} + (1-\alpha_i)\frac{\mathrm{d}\ln L_i/Y}{\mathrm{d}\ln w/r} &= \frac{\mathrm{d}\ln K_i/Y}{\mathrm{d}\ln w/r} - (1-\alpha_i)\frac{\mathrm{d}\ln K_i/L_i}{\mathrm{d}\ln w/r} \\
&= \frac{\mathrm{d}\ln K_i/Y}{\mathrm{d}\ln w/r} - (1-\alpha_i)\frac{\frac{1}{\varepsilon}\frac{\mathrm{d}\ln K_i/Y}{\mathrm{d}\ln w/r}+\alpha}{(1-\tilde{\alpha}_i)\frac{1}{\varepsilon}+\tilde{\alpha}_i\frac{1}{\sigma}} \\
&= \varepsilon\left(\frac{1}{\bar{b}}-\alpha\right) - \frac{1}{\bar{b}}\frac{(1-\alpha_i)}{(1-\tilde{\alpha}_i)\frac{1}{\varepsilon}+\tilde{\alpha}_i\frac{1}{\sigma}} \\
&= \varepsilon\left(\frac{b_i}{\bar{b}}\alpha_i-\alpha\right)
\end{aligned}$$

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With this, we can derive an expression for the aggregate elasticity of substitution. Note that the share of plants in the cross section whose current choice of capital was determined j periods ago is Γ_j. Further, among those plants, the distribution of technology states j periods ago is simply the stationary distribution of technology, $T(\tau)$. Letting $T_j(\tau_j;\tau_0)$ be the j-steps ahead conditional distribution of technology, we can use (I.2) to express the difference between our baseline estimate and the true elasticity, $\hat{\sigma}^{agg}-\sigma^{agg}$, is

$$\sum_{j=0}^{\infty}\Gamma_j\int\int\frac{(\alpha(\tau_j;\tau_0)-\alpha)\theta(\tau_j;\tau_0)}{\alpha(1-\alpha)}\left\{(\alpha(\tau_j;\tau_0)-\alpha)\varepsilon-\left(\frac{b(\tau_j;\tau_0)}{\bar{b}(\tau_0)}\alpha(\tau_j;\tau_0)-\alpha\right)\varepsilon\right\}dT_j(\tau_j;\tau_0)\mathrm{d}T(\tau_0)$$

This can be rearranged as

$$\int\mathbb{E}\left[\sum_{j=0}^{\infty}\Gamma_j\frac{(\alpha(\tau_j;\tau_0)-\alpha)\theta(\tau_j;\tau_0)}{\alpha(1-\alpha)}\left\{(\alpha(\tau_j;\tau_0)-\alpha)\varepsilon-\left(\frac{b(\tau_j;\tau_0)}{\bar{b}(\tau_0)}\alpha(\tau_j;\tau_0)-\alpha\right)\varepsilon\right\}\middle|\,\tau_0\right]\mathrm{d}T(\tau_0)$$

Or rearranged further as

$$\hat{\sigma}^{agg}-\sigma^{agg}=\varepsilon\int\mathbb{E}\left[\sum_{j=0}^{\infty}\Gamma_j(\alpha(\tau_j;\tau_0)-\alpha)\left(1-\frac{b(\tau_j;\tau_0)}{\bar{b}(\tau_0)}\right)\frac{\alpha(\tau_j;\tau_0)\theta(\tau_j;\tau_0)}{\alpha(1-\alpha)}\middle|\,\tau_0\right]\mathrm{d}T(\tau_0)$$

Since $\alpha(\tau_j;\tau_0)\theta(\tau_j;\tau_0)=\frac{rK(\tau_0)}{rK+wL}$, this is

$$\hat{\sigma}^{agg}-\sigma^{agg}=\varepsilon\int\mathbb{E}\left[\sum_{j=0}^{\infty}\Gamma_j(\alpha(\tau_j;\tau_0)-\alpha)\left(1-\frac{b(\tau_j;\tau_0)}{\bar{b}(\tau_0)}\right)\middle|\,\tau_0\right]\frac{rK(\tau_0)}{rK+wL}\frac{1}{\alpha(1-\alpha)}\mathrm{d}T(\tau_0)$$

Finally, since $1=E\left[\sum_{j=0}^{\infty}\Gamma_j\frac{b(\tau_j;\tau_0)}{\bar{b}(\tau_0)}\middle|\,\tau_0\right]$, we have

$$\hat{\sigma}^{agg}-\sigma^{agg}=\varepsilon\int\mathbb{E}\left[\sum_{j=0}^{\infty}\Gamma_j(\alpha(\tau_j;\tau_0)-\bar{\alpha}(\tau_0))\left(1-\frac{b(\tau_j;\tau_0)}{\bar{b}(\tau_0)}\right)\middle|\,\tau_0\right]\frac{rK(\tau_0)}{rK+wL}\frac{1}{\alpha(1-\alpha)}\mathrm{d}T(\tau_0)$$

Defining $x(\tau)\equiv\mathbb{E}\left[\sum_{j=0}^{\infty}\Gamma_j(\alpha(\tau_j;\tau_0)-\bar{\alpha}(\tau_0))\left(\frac{b(\tau_j;\tau_0)}{\bar{b}(\tau_0)}-1\right)\middle|\,\tau_0\right]$ to be the probability-weighted covariance of $\alpha(\tau_j;\tau_0)$ and $\frac{b(\tau_j;\tau_0)}{b(\tau_0)}$ within a spell of non-adjustment, we can express the bias as

$$\hat{\sigma}^{agg} - \sigma^{agg} = \varepsilon \int [-x(\tau_0)] \frac{rK(\tau_0)}{rK + wL} \frac{1}{\alpha(1-\alpha)} \mathrm{d}T(\tau_0) \tag{I.11}$$

Sign of the Bias In this section, we argue that in the presence of adjustment frictions to capital, the true aggregate elasticity is likely to be lower than our baseline estimate. If we could observe $\tilde{\alpha}(\tau_j, \tau_0)$, then could measure σ^{agg} directly. However, in the presence of adjustment frictions, we cannot back out technology from observed cost shares. Nevertheless, we will show that, in the empirically relevant case of $\varepsilon > 1 > \sigma$, the terms $x(\tau)$, which measure the covariance of $\alpha(\tau_j;\tau_0)$ and $b(\tau_j;\tau_0) \equiv \frac{\left(\frac{\varepsilon}{\sigma}-1\right)\frac{\tilde{\alpha}(\tau_j;\tau_0)}{\alpha(\tau_j;\tau_0)}+1}{\left(\frac{\varepsilon}{\sigma}-1\right)\tilde{\alpha}(\tau_j;\tau_0)+1}$ during the spell of non-adjustment, are likely to be negative.

First, any changes in i's capital-labor ratio pushes $\alpha()$ and $b()$ in opposite directions. α is increasing in the capital-labor ratio and, since $\sigma < 1$, $\tilde{\alpha}$ is decreasing in the capital-labor ratio. Since b is decreasing in α and increasing in $\tilde{\alpha}$, b is decreasing in the capital-labor ratio. The only possible countervailing force that can cause $x(\tau)$ to be positive is changes in the bias of technology.

An important consequence is that if the main forces affecting a plant during a spell of non-adjustment are demand shocks or Hicks-neutral productivity shocks, the covariance will be unambiguously negative for all τ. All of these forces leave the bias of technology unchanged.

We next examine non-neutral shocks. In this case, it is possible that $x(\tau)$ will be positive for some τ, but there are still several forces that push α and b in opposite directions. First, b is directly decreasing in α, which leads to a mechanical negative relationship. Second, the changes in L in response to any shock push α and $\tilde{\alpha}$ in opposite directions. For $x(\tau)$ to be positive, the direct impact of the change in the bias of technology on $\tilde{\alpha}$ must be so large that it dominates these two forces.

I.5 A second order approximation with Misallocation

In this section we characterize the difference between the true aggregate elasticity and our baseline estimate using a second order approximation. We let variables with the superscript $*$ denote the value in the allocation without adjustment frictions. In addition, we define $\kappa \equiv \left(\frac{\varepsilon}{\sigma} - 1\right)$.

Abusing notation, we can express L, α, $\tilde{\alpha}$ and b in terms of the technology at the beginning of the spell, τ, and the current technology, A, B. For example, given that capital depends only on technology at the beginning of the spell, $L(A, B; \tau)$ satisfies the first order condition

$$\frac{\varepsilon - 1}{\varepsilon} PY^{1/\varepsilon} \left[(AK(\tau))^{\frac{\sigma-1}{\sigma}} + (BL(A,B;\tau))^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}\frac{\varepsilon-1}{\varepsilon}-1} B^{\frac{\sigma-1}{\sigma}} L(A,B;\tau)^{-\frac{1}{\sigma}} = w \tag{I.12}$$

With this, we have

$$\begin{aligned} \alpha(A,B;\tau) &= \frac{rK(\tau)}{rK(\tau) + wL(A,B;\tau)} \\ \tilde{\alpha}(A,B;\tau) &= \frac{(AK(\tau))^{\frac{\sigma-1}{\sigma}}}{(AK(\tau))^{\frac{\sigma-1}{\sigma}} + (BL(A,B;\tau))^{\frac{\sigma-1}{\sigma}}} \\ b(A,B;\tau) &= \frac{\kappa\frac{\tilde{\alpha}(A,B;\tau)}{\alpha(A,B;\tau)} + 1}{\kappa\tilde{\alpha}(A,B;\tau) + 1} \end{aligned}$$

We begin with a lemma characterizing how changes in technology within a spell of non-adjustment affect labor used.

Lemma I.2 *Within a spell of non-adjustment starting with technology τ,*

$$\begin{aligned}\frac{d\ln L(A,B;\tau)}{d\ln A} &= 1-\frac{1}{\kappa\tilde{\alpha}(A,B;\tau)+1}\\ \frac{d\ln L(A,B;\tau)}{d\ln B} &= \frac{\varepsilon}{\kappa\tilde{\alpha}(A,B;\tau)+1}-1\end{aligned}$$

Proof. Taking logs and differentiating (I.12) with respect to A and B respectively gives

$$\begin{aligned}\left(\frac{\sigma}{\sigma-1}\frac{\varepsilon-1}{\varepsilon}-1\right)\frac{(AK)^{\frac{\sigma-1}{\sigma}}\frac{\sigma-1}{\sigma}+(BL)^{\frac{\sigma-1}{\sigma}}\frac{\sigma-1}{\sigma}\frac{d\ln L}{d\ln A}}{(AK)^{\frac{\sigma-1}{\sigma}}+(BL)^{\frac{\sigma-1}{\sigma}}}-\frac{1}{\sigma}\frac{d\ln L}{d\ln A} &= 0\\ \left(\frac{\sigma}{\sigma-1}\frac{\varepsilon-1}{\varepsilon}-1\right)\frac{(BL)^{\frac{\sigma-1}{\sigma}}\frac{\sigma-1}{\sigma}\left(1+\frac{d\ln L}{d\ln B}\right)}{(AK)^{\frac{\sigma-1}{\sigma}}+(BL)^{\frac{\sigma-1}{\sigma}}}+\frac{\sigma-1}{\sigma}-\frac{1}{\sigma}\frac{d\ln L}{d\ln B} &= 0\end{aligned}$$

Using $\tilde{\alpha}=\frac{(AK)^{\frac{\sigma-1}{\sigma}}}{(AK)^{\frac{\sigma-1}{\sigma}}+(BL)^{\frac{\sigma-1}{\sigma}}}$ and simplifying gives

$$\begin{aligned}\left(\frac{1}{\sigma}-\frac{1}{\varepsilon}\right)\left(\tilde{\alpha}+(1-\tilde{\alpha})\frac{d\ln L}{d\ln A}\right)-\frac{1}{\sigma}\frac{d\ln L}{d\ln A} &= 0\\ \left(\frac{1}{\sigma}-\frac{1}{\varepsilon}\right)(1-\tilde{\alpha})\left(1+\frac{d\ln L}{d\ln B}\right)+\frac{\sigma-1}{\sigma}-\frac{1}{\sigma}\frac{d\ln L}{d\ln B} &= 0\end{aligned}$$

Solving for $\frac{d\ln L}{d\ln A}$ and $\frac{d\ln L}{d\ln B}$ respectively gives

$$\begin{aligned}\frac{\kappa\tilde{\alpha}}{\kappa\tilde{\alpha}+1} &= \frac{d\ln L}{d\ln A}\\ \frac{\varepsilon}{\kappa\tilde{\alpha}+1}-1 &= \frac{d\ln L}{d\ln B}\end{aligned}$$

■

We now use these results to derive a second order approximation of

$$\sigma^{agg}-\hat{\sigma}^{agg}=\int x(\tau)\frac{rK(\tau)}{rK+wL}dT(\tau)$$

We parameterize an economy by ν. In particular, we hold fixed the cross sectional distribution of technology $T(\tau)$, but use the perturbation parameter ν to parameterize the within-spell variation in technology. In particular, If A_{τ_0} and B_{τ_0} are the technologies at the beginning of the spell, we let $A(\nu,\tau_j,\tau_0)\equiv A_{\tau_0}^{1-\nu}A_{\tau_j}^{\nu}$ and $B(\nu,\tau_j,\tau_0)\equiv B_{\tau_0}^{1-\nu}B_{\tau_j}^{\nu}$. Thus in an economy with $\nu=0$, technology remains fixed within a spell, whereas an economy with $\nu=1$ is our baseline. Nevertheless, because technologies are uncorrelated with spells of adjustment, the cross-sectional distribution $T(\tau)$ is the same in both economies.

In an economy ν, the difference between the true aggregate elasticity and our baseline estimate would be

$$\sigma^{agg}(\nu)-\hat{\sigma}^{agg}(\nu)=\int x(\tau;\nu)\frac{rK(\tau;\nu)}{rK(\nu)+wL(\nu)}dT(\tau)$$

We will take a second order expansion of $\sigma^{agg} - \hat{\sigma}^{agg} \equiv \sigma^{agg}(1) - \hat{\sigma}^{agg}(1)$ around $\nu = 0$. The following lemmas will be helpful in evaluating that expansion.

Lemma I.3 *Define where* $H(\tau) \equiv \frac{(1-\alpha^*(\tau))\frac{1}{\sigma}+\alpha^*(\tau)(\kappa+1)}{(\kappa\alpha^*(\tau)+1)^2}$

$$\begin{aligned}
\left.\frac{d\alpha(A,B;\tau)}{d\ln A}\right|_{\nu=0} &= -\alpha^*(\tau)\left(1-\alpha^*(\tau)\right)\frac{\kappa\alpha^*(\tau)}{\kappa\alpha^*(\tau)+1} \\
\left.\frac{d\alpha(A,B;\tau)}{d\ln B}\right|_{\nu=0} &= \alpha^*(\tau)\left(1-\alpha^*(\tau)\right)\left(1-\frac{\varepsilon}{\kappa\alpha^*(\tau)+1}\right) \\
\left.\frac{d\ln b(A,B;\tau)}{d\ln A}\right|_{\nu=0} &= \left(1-\alpha^*(\tau)\right)\frac{\kappa}{\kappa+1}\left(1-H(\tau)\right) \\
\left.\frac{d\ln b(A,B;\tau)}{d\ln B}\right|_{\nu=0} &= -\left(1-\alpha^*(\tau)\right)\frac{\kappa}{\kappa+1}\left(1-\varepsilon H(\tau)\right)
\end{aligned}$$

Proof. To find the expressions for $\frac{d\ln\frac{\alpha(A,B;\tau)}{1-\alpha(A,B;\tau)}}{d\ln A}$, and $\frac{d\ln\frac{\alpha(A,B;\tau)}{1-\alpha(A,B;\tau)}}{d\ln B}$, we can simply differentiate with respect to A and B. Suppressing the arguments $(A,B;\tau)$, these are

$$\begin{aligned}
\frac{d\ln\frac{\alpha}{1-\alpha}}{d\ln A} &= \frac{d\ln\frac{rK}{wL}}{d\ln A} = -\frac{d\ln L}{d\ln A} = -\frac{\kappa\tilde{\alpha}}{\kappa\tilde{\alpha}+1} \\
\frac{d\ln\frac{\alpha}{1-\alpha}}{d\ln B} &= \frac{d\ln\frac{rK}{wL}}{d\ln B} = -\frac{d\ln L}{d\ln B} = 1-\frac{\varepsilon}{\kappa\tilde{\alpha}+1}
\end{aligned}$$

The results follow from $d\alpha = \alpha(1-\alpha)\, d\ln\frac{\alpha}{1-\alpha}$. Similarly, to find expressions for $\frac{d\ln\frac{\tilde{\alpha}}{1-\tilde{\alpha}}}{d\ln A}$ and $\frac{d\ln\frac{\tilde{\alpha}}{1-\tilde{\alpha}}}{d\ln B}$ we can simply differentiate:

$$\begin{aligned}
\frac{d\ln\frac{\tilde{\alpha}}{1-\tilde{\alpha}}}{d\ln A} &= \frac{d\ln\left(\frac{AK}{BL}\right)^{\frac{\sigma-1}{\sigma}}}{d\ln A} = \frac{\sigma-1}{\sigma}\left(1-\frac{d\ln L}{d\ln A}\right) \\
\frac{d\ln\frac{\tilde{\alpha}}{1-\tilde{\alpha}}}{d\ln B} &= \frac{d\ln\left(\frac{AK}{BL}\right)^{\frac{\sigma-1}{\sigma}}}{d\ln B} = \frac{\sigma-1}{\sigma}\left(-1-\frac{d\ln L}{d\ln B}\right)
\end{aligned}$$

These can be used to derive expressions for $\frac{d\ln b_i}{d\ln A}$ and $\frac{d\ln b_i}{d\ln B}$

$$\begin{aligned}
\frac{d\ln b}{d\ln A} &= \frac{d\ln\frac{\kappa\frac{\tilde{\alpha}}{\alpha}+1}{\kappa\tilde{\alpha}+1}}{d\ln A} = \frac{\kappa\frac{\tilde{\alpha}}{\alpha}}{\kappa\frac{\tilde{\alpha}}{\alpha}+1}\left(\frac{d\ln\tilde{\alpha}}{d\ln A}-\frac{d\ln\alpha}{d\ln A}\right)-\frac{\kappa\tilde{\alpha}\frac{d\ln\tilde{\alpha}}{d\ln A}}{\kappa\tilde{\alpha}+1} \\
&= \frac{\kappa\frac{\tilde{\alpha}}{\alpha}}{\kappa\frac{\tilde{\alpha}}{\alpha}+1}\left(\left(1-\tilde{\alpha}\right)\frac{\sigma-1}{\sigma}\left(1-\frac{d\ln L}{d\ln A}\right)-\left(1-\alpha\right)\left(-\frac{d\ln L}{d\ln A}\right)\right) \\
&\quad -\frac{\kappa\tilde{\alpha}}{\kappa\tilde{\alpha}+1}\left(1-\tilde{\alpha}\right)\frac{\sigma-1}{\sigma}\left(1-\frac{d\ln L}{d\ln A}\right)
\end{aligned}$$

Evaluating this at $\nu = 0$ gives

$$\begin{aligned}
\left.\frac{d\ln b}{d\ln A}\right|_{\nu=0} &= (1-\alpha^*)\left[\frac{\kappa}{\kappa+1}\left(\frac{\sigma-1}{\sigma}\left(1-\frac{d\ln L}{d\ln A}\right)-\left(-\frac{d\ln L}{d\ln A}\right)\right)-\frac{\kappa\alpha^*}{\kappa\alpha^*+1}\frac{\sigma-1}{\sigma}\left(1-\frac{d\ln L}{d\ln A}\right)\right] \\
&= (1-\alpha^*)\frac{\kappa}{\kappa+1}\left[\left(\frac{\sigma-1}{\sigma}\left(1-\frac{d\ln L}{d\ln A}\right)-\left(-\frac{d\ln L}{d\ln A}\right)\right)-\frac{(\kappa+1)\,\alpha^*}{\kappa\alpha^*+1}\frac{\sigma-1}{\sigma}\left(1-\frac{d\ln L}{d\ln A}\right)\right] \\
&= (1-\alpha^*)\frac{\kappa}{\kappa+1}\left[1-\left\{\frac{(1-\alpha^*)\frac{1}{\sigma}+\alpha^*(\kappa+1)}{(\kappa\alpha^*+1)}\right\}\left(1-\frac{d\ln L}{d\ln A}\right)\right] \\
&= (1-\alpha^*)\frac{\kappa}{\kappa+1}\left[1-H(\tau)\right]
\end{aligned}$$

Similarly, we can differentiate with respect to B

$$\begin{aligned}
\frac{d\ln b}{d\ln B} &= \frac{d\ln\frac{\kappa\frac{\tilde{\alpha}}{\alpha}+1}{\kappa\tilde{\alpha}+1}}{d\ln B} = \frac{\kappa\frac{\tilde{\alpha}}{\alpha}}{\kappa\frac{\tilde{\alpha}}{\alpha}+1}\left(\frac{d\ln\tilde{\alpha}}{d\ln B}-\frac{d\ln\alpha}{d\ln B}\right)-\frac{\kappa\tilde{\alpha}\frac{d\ln\tilde{\alpha}}{d\ln B}}{\kappa\tilde{\alpha}+1} \\
&= \frac{\kappa\frac{\tilde{\alpha}}{\alpha}}{\kappa\frac{\tilde{\alpha}}{\alpha}+1}\left((1-\tilde{\alpha})\frac{\sigma-1}{\sigma}\left(-1-\frac{d\ln L}{d\ln B}\right)-(1-\alpha)\left(-\frac{d\ln L}{d\ln B}\right)\right) \\
&\quad -(1-\tilde{\alpha})\frac{\kappa\tilde{\alpha}}{\kappa\tilde{\alpha}+1}\frac{\sigma-1}{\sigma}\left(-1-\frac{d\ln L}{d\ln B}\right)
\end{aligned}$$

Evaluating this at $\nu = 0$ gives

$$\begin{aligned}
\left.\frac{d\ln b}{d\ln B}\right|_{\nu=0} &= (1-\alpha^*)\left\{\frac{\kappa}{\kappa+1}\left(\frac{\sigma-1}{\sigma}\left(-1-\frac{d\ln L}{d\ln B}\right)-\left(-\frac{d\ln L}{d\ln B}\right)\right)-\frac{\kappa\alpha^*}{\kappa\alpha^*+1}\frac{\sigma-1}{\sigma}\left(-1-\frac{d\ln L}{d\ln B}\right)\right\} \\
&= (1-\alpha^*)\frac{\kappa}{\kappa+1}\left\{\left(\frac{\sigma-1}{\sigma}\left(-1-\frac{d\ln L}{d\ln B}\right)-\left(-\frac{d\ln L}{d\ln B}\right)\right)-\frac{(\kappa+1)\,\alpha^*}{\kappa\alpha^*+1}\frac{\sigma-1}{\sigma}\left(-1-\frac{d\ln L}{d\ln B}\right)\right\} \\
&= -(1-\alpha^*)\frac{\kappa}{\kappa+1}\left\{1-\left[\frac{(1-\alpha^*)\frac{1}{\sigma}+\alpha^*(\kappa+1)}{\kappa\alpha^*+1}\right]\left(1+\frac{d\ln L}{d\ln B}\right)\right\} \\
&= -(1-\alpha^*)\frac{\kappa}{\kappa+1}\left\{1-\varepsilon H(\tau)\right\}
\end{aligned}$$

■

Lemma I.4 *Let* $V(\tau) \equiv \sum_{j=0}^{\infty}\Gamma_j\mathbb{E}\left[\left(\begin{array}{c}\ln A_{\tau_j}-\overline{\ln A}(\tau)\\ \ln B_{\tau_j}-\overline{\ln B}(\tau)\end{array}\right)\left(\begin{array}{c}\ln A_{\tau_j}-\overline{\ln A}(\tau)\\ \ln B_{\tau_j}-\overline{\ln B}(\tau)\end{array}\right)'\middle|\,\tau\right]$ *be the probability-weighted variance-covariance matrix of* $(\log A, \log B)$ *over the horizon of the spell.*

$$\begin{aligned}
\left.x(\tau;\nu)\right|_{\nu=0} &= 0 \\
\left.\frac{\partial x(\tau,\nu)}{\partial\nu}\right|_{\nu=0} &= 0 \\
\left.\frac{1}{2}\frac{\partial^2 x(\tau,\nu)}{\partial\nu^2}\right|_{\nu=0} &= \frac{\kappa\alpha^*(\tau)\left(1-\alpha^*(\tau)\right)^2}{(\kappa+1)\left(\kappa\alpha^*(\tau)+1\right)}\left(\begin{array}{c}-\kappa\alpha^*(\tau)\\ \kappa\alpha^*(\tau)+1-\varepsilon\end{array}\right)'V(\tau)\left(\begin{array}{c}1-H(\tau)\\ -1+\varepsilon H(\tau)\end{array}\right)
\end{aligned}$$

Proof. The definition of $x(\tau;\nu)$ along with is first and second derivatives with respect to ν are

$$\begin{aligned}
x(\tau;\nu) &= \sum_{j=0}^{\infty}\Gamma_j\mathbb{E}\left[\left(\alpha(\tau_j;\tau,\nu)-\bar{\alpha}(\tau,\nu)\right)\left(\frac{b(\tau_j;\tau,\nu)}{\bar{b}(\tau;\nu)}-1\right)\right] \\
\frac{dx(\tau;\nu)}{d\nu} &= \sum_{j=0}^{\infty}\Gamma_j\mathbb{E}\left[\frac{d\left[\alpha(\tau_j;\tau,\nu)-\bar{\alpha}(\tau,\nu)\right]}{d\nu}\left(\frac{b(\tau_j;\tau,\nu)}{\bar{b}(\tau;\nu)}-1\right)+\left(\alpha(\tau_j;\tau,\nu)-\bar{\alpha}(\tau,\nu)\right)\frac{d\left(\frac{b(\tau_j;\tau,\nu)}{\bar{b}(\tau;\nu)}\right)}{d\nu}\right] \\
\frac{d^2x(\tau;\nu)}{d\nu^2} &= \sum_{j=0}^{\infty}\Gamma_j\mathbb{E}\left[\begin{array}{c}\frac{d^2\left[\alpha(\tau_j;\tau,\nu)-\bar{\alpha}(\tau,\nu)\right]}{d\nu^2}\left(\frac{b(\tau_j;\tau,\nu)}{\bar{b}(\tau;\nu)}-1\right)\\ +2\frac{d\left[\alpha(\tau_j;\tau,\nu)-\bar{\alpha}(\tau,\nu)\right]}{d\nu}\frac{d\left(\frac{b(\tau_j;\tau,\nu)}{\bar{b}(\tau;\nu)}\right)}{d\nu}\\ +\left(\alpha(\tau_j;\tau,\nu)-\bar{\alpha}(\tau,\nu)\right)\frac{d^2\left(\frac{b(\tau_j;\tau,\nu)}{\bar{b}(\tau;\nu)}\right)}{d\nu^2}\end{array}\right]
\end{aligned}$$

Since $\sum_{j=0}^{\infty}\Gamma_j\mathbb{E}\left[\alpha^*(\tau_j;\tau,\nu)-\bar{\alpha}^*(\tau,\nu)\right]=\sum_{j=0}^{\infty}\Gamma_j\mathbb{E}\left[\frac{b(\tau_j;\tau,\nu)}{\bar{b}(\tau;\nu)}-1\right]=0$, we immediately have that $x(\tau;\nu)|_{\nu=0}=\left.\frac{\partial x(\tau,\nu)}{\partial\nu}\right|_{\nu=0}=0$, and

$$\frac{1}{2}\left.\frac{d^2x(\tau;\nu)}{d\nu^2}\right|_{\nu=0}=\sum_{j=0}^{\infty}\Gamma_j\mathbb{E}\left[\left.\frac{d\left[\alpha(\tau_j;\tau,\nu)-\bar{\alpha}(\tau,\nu)\right]}{d\nu}\frac{d\left(\frac{b(\tau_j;\tau,\nu)}{\bar{b}(\tau;\nu)}\right)}{d\nu}\right|_{\nu=0}\right]$$

To get at each of these expressions, we have

$$\begin{aligned}
\left.\frac{d\left[\alpha(\tau_j;\tau,\nu)-\bar{\alpha}(\tau,\nu)\right]}{d\nu}\right|_{\nu=0} &= \left.\frac{d\alpha(\tau_j;\tau,\nu)}{d\ln A}\right|_{\nu=0}\ln\frac{A_{\tau_j}}{A_\tau}+\left.\frac{d\alpha(\tau_j;\tau,\nu)}{d\ln A}\right|_{\nu=0}\ln\frac{B_{\tau_j}}{B_\tau} \\
&\quad -\sum_{j=0}^{\infty}\Gamma_j\mathbb{E}\left[\left.\frac{d\alpha(\tau_j;\tau,\nu)}{d\ln A}\right|_{\nu=0}\ln\frac{A_{\tau_j}}{A_\tau}+\left.\frac{d\alpha(\tau_j;\tau,\nu)}{d\ln A}\right|_{\nu=0}\ln\frac{B_{\tau_j}}{B_\tau}\right] \\
&= \left.\frac{d\alpha(\tau_j;\tau,\nu)}{d\ln A}\right|_{\nu=0}\left(\ln A_{\tau_j}-\overline{\ln A}(\tau)\right)+\left.\frac{d\alpha(\tau_j;\tau,\nu)}{d\ln A}\right|_{\nu=0}\left(\ln B_{\tau_j}-\overline{\ln B}(\tau)\right) \\
&= \left(\begin{array}{c}\left.\frac{d\alpha(\tau_j;\tau,\nu)}{d\ln A}\right|_{\nu=0}\\ \left.\frac{d\alpha(\tau_j;\tau,\nu)}{d\ln B}\right|_{\nu=0}\end{array}\right)'\left(\begin{array}{c}\ln A_{\tau_j}-\overline{\ln A}(\tau)\\ \ln B_{\tau_j}-\overline{\ln B}(\tau)\end{array}\right)
\end{aligned}$$

Using the expression for In addition, we have

$$\begin{aligned}
\left.\frac{d\left(\frac{b(\tau_j;\tau,\nu)}{\bar{b}(\tau;\nu)}\right)}{d\nu}\right|_{\nu=0} &= \left.\frac{b(\tau_j;\tau,\nu)}{\bar{b}(\tau;\nu)}\right|_{\nu=0}\left.\left(\frac{d\ln b(\tau_j;\tau,\nu)}{d\nu}-\frac{1}{\bar{b}(\tau;\nu)}\frac{d\bar{b}(\tau;\nu)}{d\nu}\right)\right|_{\nu=0} \\
&= \left.\frac{b(\tau_j;\tau,\nu)}{\bar{b}(\tau;\nu)}\right|_{\nu=0}\left.\left(\frac{d\ln b(\tau_j;\tau,\nu)}{d\nu}-\frac{1}{\bar{b}(\tau;\nu)}\sum_{j=0}^{\infty}\Gamma_j\mathbb{E}\left[\frac{db(\tau_j;\tau,\nu)}{d\nu}\right]\right)\right|_{\nu=0} \\
&= \left.\frac{b(\tau_j;\tau,\nu)}{\bar{b}(\tau;\nu)}\right|_{\nu=0}\left.\left(\frac{d\ln b(\tau_j;\tau,\nu)}{d\nu}-\sum_{j=0}^{\infty}\Gamma_j\mathbb{E}\left[\frac{b(\tau_j;\tau,\nu)}{\bar{b}(\tau;\nu)}\frac{d\ln b(\tau_j;\tau,\nu)}{d\nu}\right]\right)\right|_{\nu=0}
\end{aligned}$$

Using $b(\tau_j;\tau,\nu)|_{\nu=0} = \bar{b}(\tau;\nu)\big|_{\nu=0}$, we have

$$\begin{aligned}
\left.\frac{d\left(\frac{b(\tau_j;\tau,\nu)}{\bar{b}(\tau;\nu)}\right)}{d\nu}\right|_{\nu=0} &= \left.\left(\frac{d\ln b(\tau_j;\tau,\nu)}{d\nu} - \sum_{j=0}^{\infty}\Gamma_j\mathbb{E}\left[\frac{d\ln b(\tau_j;\tau,\nu)}{d\nu}\right]\right)\right|_{\nu=0} \\
&= \left.\frac{d\ln b(\tau_j;\tau,\nu)}{d\ln A}\right|_{\nu=0}\ln\frac{A_{\tau_j}}{A_\tau} + \left.\frac{d\ln b(\tau_j;\tau,\nu)}{d\ln A}\right|_{\nu=0}\ln\frac{B_{\tau_j}}{B_\tau} \\
&\quad - \sum_{j=0}^{\infty}\Gamma_j\mathbb{E}\left[\left.\frac{d\ln b(\tau_j;\tau,\nu)}{d\ln A}\right|_{\nu=0}\ln\frac{A_{\tau_j}}{A_\tau} + \left.\frac{d\ln b(\tau_j;\tau,\nu)}{d\ln A}\right|_{\nu=0}\ln\frac{B_{\tau_j}}{B_\tau}\right] \\
&= \left.\frac{d\ln b(\tau_j;\tau,\nu)}{d\ln A}\right|_{\nu=0}\left(\ln\frac{A_{\tau_j}}{A_\tau} - \overline{\ln A}(\tau)\right) + \left.\frac{d\ln b(\tau_j;\tau,\nu)}{d\ln A}\right|_{\nu=0}\left(\ln B_{\tau_j} - \overline{\ln B}(\tau)\right) \\
&= \begin{pmatrix} \ln A_{\tau_j} - \overline{\ln A}(\tau) \\ \ln B_{\tau_j} - \overline{\ln B}(\tau) \end{pmatrix}' \begin{pmatrix} \left.\frac{d\ln b(\tau_j;\tau,\nu)}{d\ln A}\right|_{\nu=0} \\ \left.\frac{d\ln b(\tau_j;\tau,\nu)}{d\ln B}\right|_{\nu=0} \end{pmatrix}
\end{aligned}$$

These, along with the definition of $V(\tau) \equiv \sum_{j=0}^{\infty}\Gamma_j\mathbb{E}\left[\begin{pmatrix} \ln A_{\tau_j} - \overline{\ln A}(\tau) \\ \ln B_{\tau_j} - \overline{\ln B}(\tau) \end{pmatrix}\begin{pmatrix} \ln A_{\tau_j} - \overline{\ln A}(\tau) \\ \ln B_{\tau_j} - \overline{\ln B}(\tau) \end{pmatrix}'\middle|\tau\right]$, we have

$$\frac{1}{2}\left.\frac{d^2x(\tau;\nu)}{d\nu^2}\right|_{\nu=0} = \begin{pmatrix} \left.\frac{d\alpha(\tau_j;\tau,\nu)}{d\ln A}\right|_{\nu=0} \\ \left.\frac{d\alpha(\tau_j;\tau,\nu)}{d\ln B}\right|_{\nu=0} \end{pmatrix}' V(\tau) \begin{pmatrix} \left.\frac{d\ln b(\tau_j;\tau,\nu)}{d\ln A}\right|_{\nu=0} \\ \left.\frac{d\ln b(\tau_j;\tau,\nu)}{d\ln B}\right|_{\nu=0} \end{pmatrix}$$

These along with the expressions from Lemma I.3 give the result. ■

We are now in position to state the main result of the section.

Proposition I.1 *For a spell of non-adjustment that begins with technology τ, let $V(\tau)$ be the probability-weighted variance-covariance matrix of $(\log A, \log B)$ over the horizon of the spell. Then, to a second order approximation*

$$\sigma^{agg} - \hat{\sigma}^{agg} = \int \begin{pmatrix} -\kappa\alpha^*(\tau) \\ \kappa\alpha^*(\tau) + 1 - \varepsilon \end{pmatrix}' V(\tau) \begin{pmatrix} 1 - H(\tau) \\ -1 + \varepsilon H(\tau) \end{pmatrix} J(\tau)dT(\tau) + O\left(\nu^3\right)$$

where $\kappa \equiv \frac{\varepsilon}{\sigma} - 1$, $H(\tau) \equiv \frac{(1-\alpha^*(\tau))\frac{1}{\sigma}+\alpha^*(\tau)(\kappa+1)}{(\kappa\alpha^*(\tau)+1)^2}$. and $J(\tau) \equiv \frac{\kappa\alpha^*(\tau)^2(1-\alpha^*(\tau))^2}{(\kappa+1)(\kappa\alpha^*(\tau)+1)}\theta^*(\tau)$.

Proof. A second order approximation yields

$$\begin{aligned}
\sigma^{agg} - \hat{\sigma}^{agg} &= \sigma^{agg}(\nu) - \hat{\sigma}^{agg}(\nu)\big|_{\nu=1} \\
&= \sigma^{agg}(\nu) - \hat{\sigma}^{agg}(\nu)\big|_{\nu=0} + \sigma^{agg\prime}(\nu) - \hat{\sigma}^{agg\prime}(\nu)\Big|_{\nu=0} + \left.\frac{\sigma^{agg\prime\prime}(\nu) - \hat{\sigma}^{agg\prime\prime}(\nu)}{2}\right|_{\nu=0} + O\left(\nu^3\right)
\end{aligned}$$

$x(\tau;\nu)|_{\nu=0} = 0$ and $\left.\frac{\partial x(\tau,\nu)}{\partial\nu}\right|_{\nu=0} = 0$ imply that the constant and first order terms are both zero.

These also imply that the second order term can be expressed as

$$\left.\frac{\sigma^{agg\prime\prime}(\nu)-\hat{\sigma}^{agg\prime\prime}(\nu)}{2}\right|_{\nu=0} = \frac{1}{2}\int\left[\begin{array}{c}\frac{\partial^2 x(\tau;\nu)}{\partial\nu^2}\frac{rK(\tau;\nu)}{rK(\nu)+wL(\nu)}+\frac{\partial x(\tau;\nu)}{\partial\nu}\frac{\partial\frac{rK(\tau;\nu)}{rK(\nu)+wL(\nu)}}{\partial\nu}\\ +x\left(\tau;\nu\right)\frac{\partial^2\frac{rK(\tau;\nu)}{rK(\nu)+wL(\nu)}}{\partial\nu^2}\end{array}\right]_{\nu=0} dT(\tau)$$

$$= \int\frac{1}{2}\left.\frac{\partial^2 x\left(\tau;\nu\right)}{\partial\nu^2}\right|_{\nu=0}\frac{rK^*}{rK^*+wL^*}dT(\tau)$$

$$= \int\frac{1}{2}\left.\frac{\partial^2 x\left(\tau;\nu\right)}{\partial\nu^2}\right|_{\nu=0}\alpha^*(\tau)\theta^*(\tau)dT(\tau)$$

The result follows using the expression for $\frac{1}{2}\left.\frac{\partial^2 x(\tau;\nu)}{\partial\nu^2}\right|_{\nu=0}$ from the previous lemma. ■

I.6 Capital Adjustment Costs: Monte Carlo

In this section we use a Monte Carlo simulation to examine the sign and magnitude of the difference between the true elasticity and our baseline estimate. We first study how the nature of the shocks facing plants determines the sign of the difference. We consider hypothetical economies:(1) Hicks neutral shocks; (2) shocks are purely labor-augmenting; (3) shocks are purely capital-augmenting; (4) shocks to A and B are perfectly negatively correlated. For each, we summarize the variation in shocks within a spell by Υ, and report the bias as a proportion of Υ. We summarize the cases as follows:

1. Shocks are Hicks-neutral: $V_{AA} = V_{BB} = V_{AB} = \Upsilon$

$$\sigma^{agg} - \hat{\sigma}^{agg} = -(\varepsilon-1)^2\Upsilon\int H(\tau)J(\tau)dT(\tau) + O(\nu^3)$$

2. Labor-augmenting shocks: $V_{BB} = \Upsilon$, $V_{AA} = V_{AB} = 0$

$$\sigma^{agg} - \hat{\sigma}^{agg} = -\Upsilon\int\left(\kappa\alpha^*(\tau)+1-\varepsilon\right)\left(1-H(\tau)\varepsilon\right)J(\tau)dT(\tau) + O(\nu^3)$$

3. Capital-augmenting shocks: $V_{AA} = \Upsilon$, $V_{BB} = V_{AB} = 0$

$$\sigma^{agg} - \hat{\sigma}^{agg} = -\Upsilon\int\kappa\alpha^*\left(\tau\right)\left(1-H(\tau)\right)J(\tau)dT(\tau) + O\left(\nu^3\right)$$

4. Negatively correlated shocks $V_{AA} = V_{BB} = -V_{AB} = \Upsilon$

$$\sigma^{agg} - \hat{\sigma}^{agg} = \Upsilon\int\left(-2\kappa\alpha^*\left(\tau\right)-\left(1-\varepsilon\right)\right)\left(2-\left(\varepsilon+1\right)H(\tau)\right)J(\tau)dT(\tau) + O\left(\nu^3\right)$$

For each, we simulate an economy with 700 locations that each contain 100 plants. We normalize the rental rate to 1 and draw the natural log of each location's wage from a uniform (0,1) distribution. We set σ to 0.34 to match the long run dynamic panel estimate using all of our instruments in an unbalanced panel (see the second column, last row of Table C.6). We set ε to 3 to match the average scale elasticity, i.e., the weighted average of the demand elasticity and the materials-primary inputs elasticity of substitution. We draw the ex-ante distribution of technology parameters A_i and B_i from a joint lognormal. We normalize the mean of A_i to 1, and choose the mean of B_i, the variances of A_i and B_i as well as their covariance to match the following four moments: an aggregate capital share of 0.3, a value of χ of 0.1, the 90-10 ratio of marginal cost across plants of 2.7, and the coefficient of a regression of $\log(\frac{\alpha_i}{1-\alpha_i})$ on $\log \theta_i$ (weighting by θ_i) of 0.08.[20]

For each of these four cases, we conduct a set of 200 simulations. In all four cases, the true elasticity is lower than the estimated elasticity. Given a value of the within-spell variance of shocks, Υ, the difference between the true elasticity and our baseline estimate is, on average across simulations, (1) $-0.017 \times \Upsilon$ assuming Hicks neutral shocks, (2) $-0.004 \times \Upsilon$ assuming labor-augmenting shocks, (3) $-0.014 \times \Upsilon$ assuming capital-augmenting shocks, $-0.020 \times \Upsilon$ assuming perfectly negatively correlated labor-augmenting and capital-augmenting shocks. We conclude that whatever the configuration of shocks, the true elasticity is likely to be lower than our baseline estimate.

We next attempt to bound the magnitude of the bias. To do this, we need to take a stand on the stochastic process driving technology differences across firms and the nature of adjustment frictions. It is likely that technology differences reflect both permanent and transitory differences. It should be clear from the previous discussion that the magnitude of the bias will be larger if the within-spell variation in technologies is larger. Thus, to find an upper bound, we examine the extreme case in which all technology differences are transitory and last for one period, i.e, technology differences are IID. Note that this is a uniform upper bound that is independent of the specification of capital adjustment frictions. Across the 200 simulations, the true elasticity is, on average, 0.026 lower than our baseline estimate.

J Additional Margins of Adjustment

This focuses on extensions of the model discussed in Section 4 of the main text. Web Appendix J.1 allows for long run shifts in the technological frontier. Web Appendix J.2 incorporates intangible capital. Web Appendix J.3 we study an environment in which each plant chooses from a menu of technologies. Lastly, Web Appendix J.4 discusses the difference between cross-sectional estimates and nationwide elasticities.

J.1 Shifts in the Technological Frontier

Shifts in factor prices may induce changes in the technological frontier, as outlined by Acemoglu (2002). Holding the technological frontier fixed, an increase in the wage would change the economy's capital-labor ratio. This would change the size of the market for innovations that complement each factor, and the subsequent adjustment of the technological frontier could amplify or dampen the initial wage increase.

We characterize the technological frontier as a set of intermediate input varieties that complement capital and a set that complement labor. The two sets can be respectively aggregated into two bundles, $M_K \equiv \left(\int_0^{N_K} M_K(j)^{\frac{\varphi-1}{\varphi}} dj\right)^{\frac{\varphi}{\varphi-1}}$ and $M_L \equiv \left(\int_0^{N_L} M_L(j)^{\frac{\varphi-1}{\varphi}} dj\right)^{\frac{\varphi}{\varphi-1}}$, so that the state of technology can be summarized by the measure of varieties of each type, N_K and N_L.

[20] Figure 5 depicts the aggregate share for the manufacturing sector over time, and Figure 1 values of χ across industries. Table 1 in Syverson (2004) examines dispersion in productivity (our value corresponds to the 90-10 ratio in TFP computed using plant specific input elasticities). Table 3 in Raval (2019) reports the coefficient of regressions of the capital share to labor share ratio on value added, weighting by value added, with estimates ranging from 0.05 to 0.09 using the Census of Manufactures across years, and 0.06 to 0.11 using the Annual Survey of Manufactures.

Plant i produces its output using capital, labor, intermediate inputs that complement capital and labor, as well as a third intermediate input that does not complement either factor. It is convenient to describe i's production function using a nested structure:

$$Y_i = F_i(Y_{Ki}, Y_{Li}, M_{0i})$$

with $Y_{Ki} \equiv K_i^{\psi} M_{Ki}^{1-\psi}$ and $Y_{Li} \equiv L_i^{\psi} M_{Li}^{1-\psi}$.

Each variety of intermediate input is produced by a monopolist by using ϱ units of the final good aggregate, so no capital or labor is used. Monopolists compete monopolistically and thus set a price of $\frac{\varphi}{\varphi-1}\varrho P$. The unit cost of the input bundle that complements factor $x \in \{K, L\}$ is thus $q_x = \left(\int_0^{N_x} \left(\frac{\varphi}{\varphi-1}\varrho P\right)^{1-\varphi} dj\right)^{\frac{1}{1-\varphi}} = \frac{\varphi}{\varphi-1}\varrho P N_x^{\frac{1}{1-\varphi}}$.

Aggregate factor shares depend on relative factor prices and the technological frontier, N_K and N_L. We now distinguish between the short-run aggregate elasticity which holds the technological frontier fixed and the long-run elasticity which includes shifts in the frontier. These two elasticities are related by

$$\underbrace{\frac{\mathrm{d}\ln\frac{\alpha}{1-\alpha}}{\mathrm{d}\ln w/r}}_{\sigma^{agg,LR}-1} = \frac{\partial\ln\frac{\alpha}{1-\alpha}}{\partial\ln N_K}\frac{\mathrm{d}\ln N_K}{\mathrm{d}\ln w/r} + \frac{\partial\ln\frac{\alpha}{1-\alpha}}{\partial\ln N_L}\frac{\mathrm{d}\ln N_L}{\mathrm{d}\ln w/r} + \underbrace{\frac{\partial\ln\frac{\alpha}{1-\alpha}}{\partial\ln w/r}}_{\sigma^{agg,SR}-1} \tag{J.1}$$

We show in Web Appendix J.1.1 that the effect of changes in the technological frontier on factor shares is characterized by

$$\frac{\partial\ln\frac{\alpha}{1-\alpha}}{\partial\ln N_K} = \frac{1}{\psi}\frac{1-\psi}{\varphi-1}\left(\sigma^{agg,SR}-1\right) \tag{J.2}$$

$$\frac{\partial\ln\frac{\alpha}{1-\alpha}}{\partial\ln N_L} = -\frac{1}{\psi}\frac{1-\psi}{\varphi-1}\left(\sigma^{agg,SR}-1\right) \tag{J.3}$$

For intuition, suppose that capital and labor are complements in the short run. The creation of varieties that complement capital reduces the relative cost of the capital aggregate Y_K, leading plants to reduce their relative expenditures on capital because Y_K and Y_L are complements.

To study shifts in the technological frontier, we specify its determinants in greater detail. A fixed mass Υ of scientists invent new varieties and license their inventions to monopolists. Scientists can direct their research toward one of the two types of intermediate varieties. If a scientist devotes effort to finding new varieties that complement $x \in \{K, L\}$, then new varieties arrive at Poisson rate γN_x^{τ}, with $\tau < 1$.[21] Existing varieties become useless at rate δ. At an interior steady state, scientists must be indifferent about devoting effort to each type of innovation. We show in Web Appendix J.1.1 that the long-run technological frontier is characterized by $N_K^{1-\tau} = \alpha\frac{\gamma\Upsilon}{\delta}$ and $N_L^{1-\tau} = (1-\alpha)\frac{\gamma\Upsilon}{\delta}$.

[21] $\tau < 0$ implies that as more varieties are discovered, new varieties are harder to find. $\tau > 0$ would capture positive spillovers from past research. $\tau = 1$ would deliver endogenous growth in the number of varieties. We abstract from growth because it would require a number of additional assumptions about how plant-level technologies and the distribution of plants evolves over time. In Web Appendix J.1.1 we study an alternative specification with spillovers across types of varieties, so that the arrival rate of new varieties that complement capital and that complement labor are $\gamma N_K^{\tau_1} N_L^{\tau_2}$ and $\gamma N_L^{\tau_1} N_K^{\tau_2}$ per unit of research respectively. We impose $\tau_1 + \tau_2 < 1$ to avoid perpetual growth. We show that the relationship between the long-run and short-run elasticities of substitution described in (J.4) below is identical with the exception that τ is replaced by $\tau_1 + \tau_2$.

Differentiating with respect to relative factor prices gives:

$$\frac{\mathrm{d}\ln N_K}{\mathrm{d}\ln w/r} = \frac{1}{1-\tau}(1-\alpha)(\sigma^{agg,LR}-1)$$
$$\frac{\mathrm{d}\ln N_L}{\mathrm{d}\ln w/r} = -\frac{1}{1-\tau}\alpha(\sigma^{agg,LR}-1).$$

Plugging these equations and (J.2) and (J.3) into (J.1) and rearranging gives

$$\sigma^{agg,LR} - 1 = \frac{1}{1+\frac{1}{\psi}\frac{1-\psi}{\varphi-1}\frac{1}{1-\tau}(1-\sigma^{agg,SR})}(\sigma^{agg,SR}-1). \tag{J.4}$$

Because $\frac{1}{\psi}\frac{1-\psi}{\varphi-1}\frac{1}{1-\tau} > 0$, if $\sigma^{agg,SR} < 1$, then $\sigma^{agg,LR}$ is between $\sigma^{agg,SR}$ and one. If $\sigma^{agg,SR} < 1$, an increase in wages initially raises the relative expenditure on labor. This induces the creation of varieties that complement labor, reducing the relative cost of the aggregate Y_{Li}. Since Y_{Li} and Y_{Ki} are complements, plants shift expenditures away from Y_{Li} and hence away from labor, dampening the initial shift in factor shares.[22]

J.1.1 Proofs for Shifts in the Technological Frontier

Claim J.1 *The elasticities of relative factor shares to the measures of varieties that complement capital and that complement labor are*

$$\frac{\partial \ln \frac{\alpha}{1-\alpha}}{\partial \ln N_K} = \frac{1}{\psi}\frac{1-\psi}{\varphi-1}\left(\sigma^{agg,SR}-1\right)$$
$$\frac{\partial \ln \frac{\alpha}{1-\alpha}}{\partial \ln N_L} = -\frac{1}{\psi}\frac{1-\psi}{\varphi-1}\left(\sigma^{agg,SR}-1\right)$$

Proof. Let P denote the aggregate price level and let $p_K \equiv \frac{1}{\psi^\psi(1-\psi)^{1-\psi}} r^\psi q_K^{1-\psi}$ and $p_L \equiv \frac{1}{\psi^\psi(1-\psi)^{1-\psi}} w^\psi q_L^{1-\psi}$ denote the respective shadow costs of Y_K and Y_L. Note first that p_K and p_L are sufficient to determine the aggregate price level, P, which solves the following fixed point problem: Given P, each plant's unit cost can be found using cost minimization $\lambda_i = \min_{Y_{Ki},Y_{Li},M_{0i}} p_K Y_{Ki} + p_L Y_{Li} + P M_{0i}$ subject to $F_i(Y_{Ki}, Y_{Li}, M_{0i}) \geq 1$. With constant markups, the price level satisfies $P^{1-\varepsilon} = \sum_i P_i^{1-\varepsilon} = \sum_i \left(\frac{\varepsilon}{\varepsilon-1}\lambda_i\right)^{1-\varepsilon}$.

As a consequence, p_K and p_L are also sufficient to characterize plant i's use of Y_{Ki} and Y_{Li}. Since $rK_i = \psi p_K Y_{Ki}$ and $wL_i = \psi p_L Y_{Li}$, p_K and p_L are sufficient to characterize α. Thus given plants' production functions, $\{F_i\}$, $\frac{\mathrm{d}\ln\frac{\alpha}{1-\alpha}}{\mathrm{d}\ln p_K/p_L}$ is well-defined.

Finally, since $\frac{p_K}{p_L} = \left(\frac{r}{w}\right)^\psi \left(\frac{q_K}{q_L}\right)^{1-\psi} = \left(\frac{r}{w}\right)^\psi \left(\frac{N_K}{N_L}\right)^{\frac{1-\psi}{1-\varphi}}$, we have that

$$\frac{\partial \ln p_K/p_L}{\partial \ln N_K} = \frac{1}{\psi}\frac{1-\psi}{\varphi-1}\frac{\partial \ln p_K/p_L}{\partial \ln w/r}$$
$$\frac{\partial \ln p_K/p_L}{\partial \ln N_L} = -\frac{1}{\psi}\frac{1-\psi}{\varphi-1}\frac{\partial \ln p_K/p_L}{\partial \ln w/r}$$

[22] Acemoglu (2003) studies a model with $\tau = 1$ and shows that long-run factor shares are fixed. While we have imposed the restriction $\tau < 1$ to abstract from growth, we can recover Acemoglu's (2003) result in the limit of $\tau \nearrow 1$.

We then have:

$$\frac{\partial \ln \frac{\alpha}{1-\alpha}}{\partial \ln N_K} = \frac{\partial \ln \frac{\alpha}{1-\alpha}}{\partial \ln p_K/p_L}\frac{\partial \ln p_K/p_L}{\partial \ln N_K} = \frac{\partial \ln \frac{\alpha}{1-\alpha}}{\partial \ln p_K/p_L}\left(\frac{1}{\psi}\frac{1-\psi}{\varphi-1}\frac{\partial \ln p_K/p_L}{\partial \ln w/r}\right) = \frac{1}{\psi}\frac{1-\psi}{\varphi-1}\frac{\partial \ln \frac{\alpha}{1-\alpha}}{\partial \ln w/r}$$
$$\frac{\partial \ln \frac{\alpha}{1-\alpha}}{\partial \ln N_L} = \frac{\partial \ln \frac{\alpha}{1-\alpha}}{\partial \ln p_K/p_L}\frac{\partial \ln p_K/p_L}{\partial \ln N_L} = \frac{\partial \ln \frac{\alpha}{1-\alpha}}{\partial \ln p_K/p_L}\left(-\frac{1}{\psi}\frac{1-\psi}{\varphi-1}\frac{\partial \ln p_K/p_L}{\partial \ln w/r}\right) = -\frac{1}{\psi}\frac{1-\psi}{\varphi-1}\frac{\partial \ln \frac{\alpha}{1-\alpha}}{\partial \ln w/r}$$

■

Claim J.2 *At an interior steady state,* $N_K^{1-\tau} = \alpha\frac{\gamma\Upsilon}{\delta}$ *and* $N_L^{1-\tau} = (1-\alpha)\frac{\gamma\Upsilon}{\delta}$.

Proof. For $x \in \{K, L\}$, let Υ_x be the mass of scientists that devote their efforts to finding new varieties that complement x. Then N_x follows the law of motion $\dot{N}_x = \Upsilon_x\gamma N_x^\tau - \delta N_x$. Let V_x be the value of the rights to a variety that complements x. This value satisfies the Bellman equation $\rho V_x = \pi_x - \delta V_x + \dot{V}_x$, where π_x is the flow profit from owning a single variety that complements x. An interior equilibrium requires that $\gamma N_K^\tau V_K = \gamma N_L^\tau V_L$. In a long-run equilibrium, V_K, V_L, N_K, N_L, π_K, and π_L are constant, which implies $V_x = \frac{\pi_x}{\rho+\delta}$, and hence

$$\gamma N_K^\tau \frac{\pi_K}{\rho+\delta} = \gamma N_L^\tau \frac{\pi_L}{\rho+\delta} \tag{J.5}$$

Noting that $\pi_x = \frac{q_x M_x}{\varphi N_x}$, we have that $\frac{\pi_K}{\pi_L} = \frac{q_K M_K/N_K}{q_L M_L/N_L}$. Since $q_K M_K = \frac{1-\psi}{\psi}rK$ and $q_L M_L = \frac{1-\psi}{\psi}wL$, we have $\frac{\pi_K}{\pi_L} = \frac{\alpha/N_K}{(1-\alpha)/N_L}$. Combining this with (J.5) and rearranging gives $\left(\frac{N_K}{N_L}\right)^{1-\tau} = \frac{\alpha}{1-\alpha}$. In steady state, the laws of motion for N_K and N_L imply $N_x^{1-\tau} = \frac{\gamma}{\delta}\Upsilon_x$, so that $\Upsilon_K = \alpha\Upsilon$ and $\Upsilon_L = (1-\alpha)\Upsilon$. Therefore $N_K^{1-\tau} = \alpha\frac{\gamma\Upsilon}{\delta}$ and $N_L^{1-\tau} = (1-\alpha)\frac{\gamma\Upsilon}{\delta}$. ■

Spillovers

Suppose that the arrival rate of new varieties depended on the existing varieties of each type, so that

$$\dot{N}_K = \gamma\Upsilon_K N_K^{\tau_1} N_L^{\tau_2} - \delta N_K$$
$$\dot{N}_L = \gamma\Upsilon_L N_L^{\tau_1} N_K^{\tau_2} - \delta N_L$$

Steady state requires $\dot{N}_K = \dot{N}_L = 0$, while an interior equilibrium requires that $\gamma N_K^{\tau_1} N_L^{\tau_2}\pi_K = \gamma N_L^{\tau_1} N_K^{\tau_2}\pi_L$. As described above, $\frac{\pi_K}{\pi_L} = \frac{\alpha/N_K}{(1-\alpha)/N_L}$. All together, these equations imply

$$N_K^{1-\tau_1-\tau_2} = \frac{\gamma\Upsilon}{\delta}\alpha^{\frac{1-\tau_1}{1-\tau_1+\tau_2}}(1-\alpha)^{\frac{\tau_2}{1-\tau_1+\tau_2}}$$
$$N_L^{1-\tau_1-\tau_2} = \frac{\gamma\Upsilon}{\delta}(1-\alpha)^{\frac{1-\tau_1}{1-\tau_1+\tau_2}}\alpha^{\frac{\tau_2}{1-\tau_1+\tau_2}}$$

The elasticities of these with respect to factor prices is therefore

$$(1-\tau_1-\tau_2)\frac{\mathrm{d}\ln N_K}{\mathrm{d}\ln w/r} = \left[(1-\alpha) - \frac{\tau_2}{1-\tau_1+\tau_2}\right](\sigma^{agg,LR}-1)$$
$$(1-\tau_1-\tau_2)\frac{\mathrm{d}\ln N_L}{\mathrm{d}\ln w/r} = \left[-\alpha - \frac{\tau_2}{1-\tau_1+\tau_2}\right](\sigma^{agg,LR}-1)$$

These along with Claim J.1 imply

$$(1-\tau_1-\tau_2)\left[\frac{\partial \ln \frac{\alpha}{1-\alpha}}{\partial \ln N_K}\frac{\mathrm{d}\ln N_K}{\mathrm{d}\ln w/r}+\frac{\partial \ln \frac{\alpha}{1-\alpha}}{\partial \ln N_L}\frac{\mathrm{d}\ln N_L}{\mathrm{d}\ln w/r}\right]=\frac{1}{\psi}\frac{1-\psi}{\varphi-1}(\sigma^{agg,SR}-1)(\sigma^{agg,LR}-1)$$

Plugging this into (J.1) and solving for the long run elasticity of substitution gives

$$\sigma^{agg,LR}-1=\frac{1}{1+\frac{1}{1-\tau_1-\tau_2}\frac{1}{\psi}\frac{1-\psi}{\varphi-1}(1-\sigma^{agg,SR})}(\sigma^{agg,SR}-1)$$

J.2 Intangible Capital

Investment in intangible capital has risen over time. In this section, we study how incorporating intangible capital into production affects both the interpretation of the elasticity of substitution between physical capital and labor and our estimation strategy.

This section takes the view that production of intangible capital is an intermediate step taken by plants in the course of producing the good that it sells to customers. The simplest version of this is a static problem in which a firm produces intangible capital using tangible inputs according to a production function $O_i = H_i(K_i^O, L_i^O, M_i^O)$ and, in turn, produces output for customers using intangible capital and more tangible inputs according to the production function $Y_i = G_i(K_i^Y, L_i^Y, M_i^Y, O_i)$. Specifically, we can write plant i's output as a function of total tangible inputs, K_i, L_i, M_i:

$$Y_i = F_i(K_i, L_i, M_i) = \max_{K_i^Y, L_i^Y, M_i^Y, K_i^O, L_i^O, M_i^O, O_i} G_i(K_i^Y, L_i^Y, M_i^Y, O_i)$$

subject to $K_i^Y + K_i^O \leq K_i$, $L_i^Y + L_i^O \leq L_i$, $M_i^Y + M_i^O \leq M_i$, and

$$O_i \leq H_i(K_i^O, L_i^O, M_i^O).$$

This formulation allows for the possibility that technical change (in either G_i or H_i) increased the role of intangible capital over time.

With this microfoundation of the plant-level production function, our baseline estimate of the aggregate elasticity of substitution between physical capital and labor recovers the correct elasticity. Our strategy recovers the elasticities of the reduced form of the indirect production function F_i, and the rest of the argument follows exactly along the lines of Section 2.

J.2.1 Marketing

There has been active discussion about whether market power—and particularly markups over marginal cost— has risen over time (De Loecker et al. (2020), Traina (2018)). As documented by Traina (2018), the share of expenditures spent on marketing and management among publicly traded firms has risen over time, so that ratios of revenue to production costs has risen more sharply than ratios of revenue to total cost. To some extent, this discussion has focused on whether marketing costs should be considered part of variable cost. In this section we describe how we incorporate marketing costs into our framework, allowing plants to invest to increase demand. As with intangible capital that increases output (studied in the last section), we show here that our baseline estimate continues to recover the aggregate elasticity, regardless of the division of expenditures between production and marketing costs, as long as these expenditures are measured.

Suppose that output is produced according to the production function $Y_i = G_i(K_i^Y, L_i^Y, M_i^Y)$ and plant i's demand depends on marketing expenditures according to the function $D_i(K_i^D, L_i^D, M_i^D)$, where the consumer's preferences can be represented by $Y = \left(\sum_i D_i^{\frac{1}{\varepsilon}} Y_i^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}$. Plant i chooses inputs for production and for marketing to maximize profit:

$$\max_{P_i, Y_i, K_i^Y, L_i^Y, M_i^Y, K_i^D, L_i^D, M_i^D} P_i Y_i - r(K_i^Y + K_i^D) - w(L_i^Y + L_i^D) - q(M_i^Y + M_i^D)$$

subject to the constraints imposed by technology $Y_i \leq G_i(K_i^Y, L_i^Y, M_i^Y)$ and consumer demand $Y_i \leq D_i(K_i^D, L_i^D, M_i^D) Y P^{\varepsilon} P_i^{-\varepsilon}$, where $P \equiv \left(D_i P_i^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}$ is the ideal price index.

With the changes of variable $\tilde{Y}_i = D^{\frac{1}{\varepsilon-1}} Y_i$ and $\tilde{P}_i = D^{-\frac{1}{\varepsilon-1}} P_i$, we can express this as

$$\max_{\tilde{P}_i, \tilde{Y}_i, K_i^Y, L_i^Y, M_i^Y, K_i^D, L_i^D, M_i^D} \tilde{P}_i \tilde{Y}_i - r(K_i^Y + K_i^D) - w(L_i^Y + L_i^D) - q(M_i^Y + M_i^D)$$

subject to $\tilde{Y}_i \leq G_i(K_i^Y, L_i^Y, M_i^Y) D_i(K_i^D, L_i^D, M_i^D)^{\frac{1}{\varepsilon-1}}$ and $\tilde{Y}_i \leq Y P^{\varepsilon} \tilde{P}_i^{-\varepsilon}$. This can be expressed even more succinctly as

$$\max_{\tilde{P}_i, \tilde{Y}_i, K_i, L_i, M_i} \tilde{P}_i \tilde{Y}_i - rK_i - wL_i - qM_i$$

subject to $\tilde{Y}_i \leq F_i(K_i, L_i, M_i)$ and $\tilde{Y}_i \leq Y P^{\varepsilon} \tilde{P}_i^{-\varepsilon}$, where $F_i(K_i, L_i, M_i)$ is a demand-adjusted production function defined as

$$F_i(K_i, L_i, M_i) \equiv \max_{K_i^Y, L_i^Y, M_i^Y, K_i^D, L_i^D, M_i^D} G_i(K_i^Y, L_i^Y, M_i^Y) D_i(K_i^D, L_i^D, M_i^D)^{\frac{1}{\varepsilon-1}}$$

subject to $K_i^Y + K_i^D \leq K_i$, $L_i^Y + L_i^D \leq L_i$, and $M_i^Y + M_i^D \leq M_i$. This is now exactly the same form as our baseline model.

For our purposes, we note that the markup over marginal cost per se is not the object of interest, and not one we use in constructing the elasticity of substitution. As discussed in Web Appendix D.3, the scale elasticity depends on both the elasticity of demand and the returns to scale of the production function F_i, and the appropriate combination will be identified by the ratio of revenue to total cost, regardless of the division of expenditures into production and marketing costs. Changes in the division of expenditures between production and marketing costs pose no additional obstacles so long as these expenditures are measured in the data.

J.2.2 Durable Intangible Capital

These arguments abstract from the fact that intangible capital is durable. While this alters the problem slightly because each plant effectively produces two kinds of output, the basic intuition carries over: each plant chooses tangible inputs to produce optimally (where production now includes investment in intangible capital). Our estimation strategy traces out how these choices change when factor prices change; the intermediate step of producing intangible capital does not fundamentally alter the problem.

When intangible capital is durable, the aggregate elasticity depends on the factor content of output relative to the factor content of intangible goods. We examine two extreme cases (producing intangible capital requires only labor or only physical capital) and one intermediate case (the factor content of intangible capital matches the factor content of output). We show that in all three cases true elasticity of substitution is quantitatively close to our baseline, with the difference ranging from 0.009 to 0.012. This happens because investment in intangible capital is a relatively small fraction of gross output (Corrado et al. (2009) and Nakamura (2010)) and estimates of depreciation rates for intangible capital tend to be relatively high (Corrado et al. (2009) and Li and Hall (2018)).

Consider an economy in which plants invest in intangible capital which serves as an additional factor of production. A plant operates two technologies, one that transforms tangible goods into an investment in intangible capital, $H_i(K_i^O, L_i^O, M_i^O)$ and one that transforms tangible and intangible inputs into output[23],

$$Y_i = G_i(K_i^Y, L_i^Y, M_i^Y)^{\phi} O'^{1-\phi}. \tag{J.6}$$

Intangible capital accumulates according to the law of motion

$$O' = H_i(K_i^O, L_i^O, M_i^O) + (1 - \delta^O)O \tag{J.7}$$

This problem corresponds to the following Bellman equation, where $V_i(O)$ is the present discounted value of i's profit when its stock of intangible capital is O:

$$V(O) = \max_{K_i^Y, L_i^Y, M_i^Y, O', K_i^O, L_i^O, M_i^O, Y_i, P_i} P_i Y_i - r(K_i^Y + K_i^O) - w(L_i^Y + L_i^O) - q(M_i^Y + M_i^O) + \beta V(O')$$

subject to the production functions (J.6), the law of motion for intangible capital (J.7), and the household's demand curve, $Y_i \leq Y(P_i/P)^{-\varepsilon}$. Let $c_i^G(r, w, q)$ and $c_i^H(r, w, q)$ be the unit cost functions associated with G_i and H_i respectively. Then the dynamic program can be simplified as

$$V(O_i) = \max_{G_i, H_i, O_i'} PY^{1/\varepsilon} \left(G_i^{\phi} O_i'^{1-\phi} \right)^{\frac{\varepsilon-1}{\varepsilon}} - c_i^G G_i - c_i^H H_i + \beta V(O_i')$$

subject to $O_i' = H_i(1 - \delta^O)O_i$. The allocation is determined by the first order and envelope conditions which, after imposing stationarity ($O_i = O_i'$), imply

$$\frac{\varepsilon - 1}{\varepsilon} \underbrace{PY^{1/\varepsilon} \left(G_i^{\phi} O_i^{1-\phi} \right)^{\frac{\varepsilon-1}{\varepsilon}}}_{P_i Y_i} = \frac{c_i^G G_i}{\phi} = \frac{[1 - \beta(1 - \delta)] c_i^H O_i}{1 - \phi}$$

and $H_i = \delta O_i$. A first key implication is that the ratio of expenditures on tangible input to revenue is

$$\frac{rK_i + wL_i + qM_i}{P_i Y_i} = \frac{c_i^G G_i + c_i^H H_i}{P_i Y_i} = \frac{c_i^G G_i + c_i^H \delta O_i}{P_i Y_i} = \frac{\varepsilon - 1}{\varepsilon} \left(\phi + \frac{\delta^O}{1 - \beta(1 - \delta^O)} (1 - \phi) \right) \tag{J.8}$$

A second key implication is that the optimal price is

$$P_i = \frac{\varepsilon}{\varepsilon - 1} \frac{1}{\phi^{\phi} (1 - \phi)^{1-\phi}} (c_i^G)^{\phi} \left[(1 - \beta(1 - \delta)) \, c_i^H \right]^{1-\phi}$$

[23] The assumption of a unitary elasticity between tangible inputs and intangible capital in the production of output is arguably a strong one. We discuss below the implications and considerations that led to this choice.

This implies that the plant's expenditure on tangible inputs relative to total expenditures is

$$z_i \equiv \frac{rK_i + wL_i + qM_i}{rK + wL + qM} = \frac{P_i Y_i}{PY} = \frac{P_i^{1-\varepsilon}}{P^{1-\varepsilon}} = \frac{\left((c_i^G)^{\phi}(c_i^H)^{1-\phi}\right)^{1-\varepsilon}}{\sum_{\tilde{i}}\left((c_{\tilde{i}}^G)^{\phi}(c_{\tilde{i}}^H)^{1-\phi}\right)^{1-\varepsilon}} \tag{J.9}$$

We are now in position to compute the aggregate elasticity of substitution. Just as in the baseline, $\sigma_i = \frac{\mathrm{d}\ln K_i/L_i}{\mathrm{d}\ln w/r}$ measures how plant i's K/L ratio changes when factor prices change. This is a reduced form elasticity that depends on how the plant substitutes between K and L in production of both intangible capital and output. Similarly, ζ_i is the reduced form elasticity that measures substitution between materials and the other primary tangible inputs. The exact arguments used in Section 2.2 imply here that the aggregate elasticity between physical capital and labor is

$$\sigma^{agg} = (1-\chi)\bar{\sigma} + \chi\bar{s}^M\bar{\zeta} + \chi(1-\bar{s}^M)\left[1 + \frac{\sum_i(\alpha_i - \alpha)\frac{\mathrm{d}\ln z_i}{\mathrm{d}\ln w/r}\theta_i}{\sum_i(\alpha_i-\alpha)^2(1-s_i^M)\theta_i}\right]$$

Our estimates of $\hat{\bar{\sigma}}$ and $\hat{\bar{\zeta}}$ match $\bar{\sigma}$ and $\bar{\zeta}$ because these are estimated directly from how changes in relative factor prices alter plants' choices of physical capital, labor, and materials. Our baseline estimate of the elasticity of substitution was computed as

$$\hat{\sigma}^{agg} = (1-\chi)\hat{\bar{\sigma}} + \chi\bar{s}^M\hat{\bar{\zeta}} + \chi(1-\bar{s}^M)\hat{\varepsilon}$$

where $\hat{\varepsilon}$ was estimated from ratios of revenue to cost, $\frac{\hat{\varepsilon}}{\hat{\varepsilon}-1} = \frac{P_iY_i}{rK_i+wL_i+qM_i}$. The difference between the true aggregate elasticity and the baseline will thus be

$$\sigma^{agg} - \hat{\sigma}^{agg} = \chi(1-\bar{s}^M)(\Upsilon - \hat{\varepsilon})$$

where $\Upsilon \equiv 1 + \frac{\sum_i(\alpha_i-\alpha)\frac{\mathrm{d}\ln z_i}{\mathrm{d}\ln w/r}\theta_i}{\sum_i(\alpha_i-\alpha)^2(1-s_i^M)\theta_i}$, or using (J.9),

$$\Upsilon \equiv 1 + (1-\varepsilon)\frac{\sum_i(\alpha_i-\alpha)\left[\phi\frac{\mathrm{d}\ln c_i^G/r}{\mathrm{d}\ln w/r} + (1-\phi)\frac{\mathrm{d}\ln c_i^H/r}{\mathrm{d}\ln w/r}\right]\theta_i}{\sum_i(\alpha_i-\alpha)^2(1-s_i^M)\theta_i}$$

Υ depends on how tangible inputs are divided between producing intangible capital and producing output directly. Unfortunately we do not observe this information. As an alternative, we study three alternatives: (i) intangible capital is produced using only capital, (ii) intangible capital is produced using only labor, and (iii) intangible capital has the same factor intensity as the production of output.

Before doing that, it will be useful to derive an expression for ε in terms of objects we can measure or calibrate. Towards this, note that the ratio of investment expenditure on intangible inputs relative to *gross* output is

$$\begin{aligned}\frac{I^O}{PY} &= \frac{\sum_i I_i^O}{PY} = \frac{\sum_i c_i^H H_i}{PY} = \frac{\sum_i \delta^O c_i^H O_i}{PY} = \frac{\sum_i \frac{\delta^O}{1-\beta(1-\delta^O)}(1-\phi)\frac{\varepsilon-1}{\varepsilon}P_iY_i}{PY} \\ &= \frac{\varepsilon-1}{\varepsilon}(1-\phi)\frac{\delta^O}{1-\beta(1-\delta^O)}\end{aligned}$$

We can then use (J.8) to write

$$
\begin{aligned}
\frac{\hat{\varepsilon}-1}{\hat{\varepsilon}} &\equiv \frac{rK_i + wL_i + qM_i}{P_iY_i} = \frac{\varepsilon-1}{\varepsilon}\left(\phi + \frac{\delta^O}{1-\beta(1-\delta^O)}(1-\phi)\right) \\
&= \frac{\varepsilon-1}{\varepsilon}\left[1 - \left(\frac{(1-\delta^O)(1-\beta)}{\delta^O}\right)\frac{\delta^O}{1-\beta(1-\delta^O)}(1-\phi)\right] \\
&= \frac{\varepsilon-1}{\varepsilon} - \frac{(1-\delta^O)(1-\beta)}{\delta^O}\frac{I^O}{PY}
\end{aligned}
$$

or more simply

$$
\frac{1}{\hat{\varepsilon}} = \frac{1}{\varepsilon} + \frac{(1-\delta^O)(1-\beta)}{\delta^O}\frac{I^O}{PY}
$$

Case 1: Intangible capital is produced using only capital

$$
\begin{aligned}
\phi\frac{\mathrm{d}\ln c_i^G/r}{\mathrm{d}\ln w/r} &= \phi\frac{wL_i^Y + (1-\alpha)qM_i^Y}{c_i^G G_i} = \frac{wL_i^Y + (1-\alpha)qM_i^Y}{\frac{\varepsilon-1}{\varepsilon}P_iY_i} = \frac{wL_i + (1-\alpha)qM_i}{\frac{\varepsilon-1}{\varepsilon}P_iY_i} \\
&= \frac{rK_i + wL_i + qM_i}{\frac{\varepsilon-1}{\varepsilon}P_iY_i}\frac{wL_i + (1-\alpha)qM_i}{rK_i + wL_i + qM_i} \\
&= \frac{\frac{\hat{\varepsilon}-1}{\hat{\varepsilon}}}{\frac{\varepsilon-1}{\varepsilon}}\left[(1-s_i^M)(1-\alpha_i) + s_i^M(1-\alpha)\right]
\end{aligned}
$$

Since $\frac{\mathrm{d}\ln c_i^H/r}{\mathrm{d}\ln w/r} = 0$ by assumption, we have

$$
\begin{aligned}
\Upsilon &= 1 + (1-\varepsilon)\frac{\sum_i(\alpha_i - \alpha)\left[\phi\frac{\mathrm{d}\ln c_i^G/r}{\mathrm{d}\ln w/r} + (1-\phi)\frac{\mathrm{d}\ln c_i^H/r}{\mathrm{d}\ln w/r}\right]\theta_i}{\sum_i(\alpha_i-\alpha)^2(1-s_i^M)\theta_i} \\
&= 1 + (\varepsilon-1)\frac{\frac{\hat{\varepsilon}-1}{\hat{\varepsilon}}}{\frac{\varepsilon-1}{\varepsilon}} = \frac{1+\varepsilon^{-1}-\hat{\varepsilon}^{-1}}{\varepsilon^{-1}} \\
&= \frac{1 - \frac{(1-\delta^O)(1-\beta)}{\delta^O}\frac{I^O}{PY}}{\hat{\varepsilon}^{-1} - \frac{(1-\delta^O)(1-\beta)}{\delta^O}\frac{I^O}{PY}}
\end{aligned}
$$

Case 2: Intangible capital is produced using only labor

$$
\begin{aligned}
\phi\frac{\mathrm{d}\ln c_i^G/r}{\mathrm{d}\ln w/r} &= \phi\frac{wL_i^Y + (1-\alpha)qM_i^Y}{c_i^G G_i} = \phi\left(1 - \frac{rK_i^Y + \alpha qM_i^Y}{c_i^G G_i}\right) \\
&= \phi - \frac{rK_i^Y + \alpha qM_i^Y}{\frac{\varepsilon-1}{\varepsilon}P_iY_i} = \phi - \frac{rK_i + \alpha qM_i}{\frac{\varepsilon-1}{\varepsilon}P_iY_i} \\
&= \phi - \frac{rK_i + wL_i + qM_i}{\frac{\varepsilon-1}{\varepsilon}P_iY_i}\frac{rK_i + \alpha qM_i}{rK_i + wL_i + qM_i} \\
&= \phi - \frac{\frac{\hat{\varepsilon}-1}{\hat{\varepsilon}}}{\frac{\varepsilon-1}{\varepsilon}}\left[(1-s_i^M)\alpha_i + s_i^M\alpha\right]
\end{aligned}
$$

Since $\frac{\mathrm{d}\ln c_i^H/r}{\mathrm{d}\ln w/r} = 1$ by assumption, we have

$$\begin{aligned}
\Upsilon &= 1+(1-\varepsilon)\frac{\sum_i(\alpha_i-\alpha)\left[\phi\frac{\mathrm{d}\ln c_i^G/r}{\mathrm{d}\ln w/r}+(1-\phi)\frac{\mathrm{d}\ln c_i^H/r}{\mathrm{d}\ln w/r}\right]\theta_i}{\sum_i(\alpha_i-\alpha)^2(1-s_i^M)\theta_i} \\
&= 1+(\varepsilon-1)\frac{\frac{\hat{\varepsilon}-1}{\hat{\varepsilon}}}{\frac{\varepsilon-1}{\varepsilon}} = \frac{1+\varepsilon^{-1}-\hat{\varepsilon}^{-1}}{\varepsilon^{-1}} \\
&= \frac{1-\frac{(1-\delta^O)(1-\beta)}{\delta^O}\frac{I^O}{PY}}{\hat{\varepsilon}^{-1}-\frac{(1-\delta^O)(1-\beta)}{\delta^O}\frac{I^O}{PY}}
\end{aligned}$$

Case 3: Production of output and intangible capital have same factor intensity

By assumption, we have

$$\frac{\mathrm{d}\ln c_i^G/r}{\mathrm{d}\ln w/r} = \frac{\mathrm{d}\ln c_i^H/r}{\mathrm{d}\ln w/r} = (1-s_i^M)(1-\alpha_i)+s_i^M(1-\alpha)$$

which implies

$$\Upsilon = 1+(1-\varepsilon)\frac{\sum_i(\alpha_i-\alpha)\left[\phi\frac{\mathrm{d}\ln c_i^G/r}{\mathrm{d}\ln w/r}+(1-\phi)\frac{\mathrm{d}\ln c_i^H/r}{\mathrm{d}\ln w/r}\right]\theta_i}{\sum_i(\alpha_i-\alpha)^2(1-s_i^M)\theta_i} = \varepsilon = \frac{1}{\hat{\varepsilon}^{-1}-\frac{(1-\delta^O)(1-\beta)}{\delta^O}\frac{I^O}{PY}}$$

Notice that in all three cases, if either the depreciation rate δ^O or the discount factor β is unity, then the baseline estimate recovers the true aggregate elasticity. If not, then we understate the true elasticity. How big might the bias be? We estimated $\hat{\varepsilon} = 4$. A variety of sources argue that investment in intangible output has grown from 5% of value added to 10% (Corrado et al. (2009) and Nakamura (2010)), which translates to less than 5% of gross output.[24] Depreciation of intangible capital is difficult to measure because we do not observe stocks or prices, estimates range from 20% − 70% (Corrado et al. (2009) and Li and Hall (2018)). To be conservative, we use $\frac{I^O}{PY} = 5\%$, set the depreciation rate to 20%, and use a discount factor of $\beta = 0.93$. These imply that the bias, $\hat{\sigma}^{agg} - \sigma^{agg} = \chi\bar{s}^M(\Upsilon - \hat{\epsilon})$ for 1987 is equal to 0.012 in cases 1 and 2, and 0.009 in case 3.

We conclude this section with some remarks about the choice of production function G_i. How exactly intangible capital enters the production function is speculative, because we have no information about the price or quantities of intangible capital. In general, the aggregate elasticity of substitution between physical capital and labor would depend on the elasticity between tangible inputs and intangible capital. Further, conditional on objects we can observe, ε would depend on a weighted average of plants' ratios of investment in intangible capital to gross output, rather than the aggregate ratio. We do not know of any data that can discipline these features. We can show, however, that even in the general case, if either $\beta = 1$ or $\delta^O = 1$, our baseline estimate recovers the true aggregate elasticity. Our calibration suggests that in practice $(1-\beta)(1-\delta^O)$ is small enough that the additional generality would be quantitatively irrelevant. Given these considerations, we imposed a unitary elasticity between tangible inputs and intangible capital to simplify the exposition in this section.

[24] Note that estimates of the ratio of intangible capital to value added are for the entire economy, so we are making the strong assumption that this ratio is the same for the manufacturing sector. In the manufacturing sector, value added was roughly 47% of gross output in 1987.

J.3 Technology Choice

In this section, we extend the model so that plants can respond to changes in factor prices by switching to an alternative technology. Using a parametric example inspired by Caselli and Coleman (2006), we show that our baseline strategy recovers the aggregate elasticity of substitution. We then generalize this result by relaxing the parametric assumptions.

A technology for plant i is a nested CES production function:

$$Y_i = \left\{ \left[(A_i K_i)^{\frac{\sigma^{within}-1}{\sigma^{within}}} + (B_i L_i)^{\frac{\sigma^{within}-1}{\sigma^{within}}} \right]^{\frac{\sigma^{within}}{\sigma^{within}-1}\frac{\zeta^{within}-1}{\zeta^{within}}} + (C_i M_i)^{\frac{\zeta^{within}-1}{\zeta^{within}}} \right\}^{\frac{\zeta^{within}}{\zeta^{within}-1}}. \quad \text{(J.10)}$$

Plant i chooses its factor-augmenting productivities A_i, B_i, C_i from a menu defined by the parameters $\mathcal{A}_i, \mathcal{B}_i, \mathcal{C}_i$:

$$\left\{ \left[(A_i/\mathcal{A}_i)^{1-\sigma^{menu}} + (B_i/\mathcal{B}_i)^{1-\sigma^{menu}} \right]^{\frac{1-\zeta^{menu}}{1-\sigma^{menu}}} + (C_i/\mathcal{C}_i)^{\frac{1}{1-\zeta^{menu}}} \right\}^{\frac{1}{1-\zeta^{menu}}} \leq 1. \quad \text{(J.11)}$$

When factor prices change, plant i can shift both its factor usage and its choice of technology. In fact, one can define the envelope of plant i's technology menu to be the output that it would produce with the optimal technology choice. For any K_i, L_i, M_i, the envelope corresponds to the choice of factor-augmenting productivities that maximize (J.10) subject to (J.11). As we show in Web Appendix J.3.1, we can solve for the envelope directly:

$$Y_i = \left\{ \left[(\mathcal{A}_i K_i)^{\frac{\sigma^{total}-1}{\sigma^{total}}} + (\mathcal{B}_i L_i)^{\frac{\sigma^{total}-1}{\sigma^{total}}} \right]^{\frac{\sigma^{total}}{\sigma^{total}-1}\frac{\zeta^{total}-1}{\zeta^{total}}} + (\mathcal{C}_i M_i)^{\frac{\zeta^{total}-1}{\zeta^{total}}} \right\}^{\frac{\zeta^{total}}{\zeta^{total}-1}} \quad \text{(J.12)}$$

where σ^{total} and ζ^{total} are defined to satisfy

$$\frac{1}{\sigma^{total}-1} = \frac{1}{\sigma^{within}-1} + \frac{1}{\sigma^{menu}-1}$$
$$\frac{1}{\zeta^{total}-1} = \frac{1}{\zeta^{within}-1} + \frac{1}{\zeta^{menu}-1}$$

σ^{within} and ζ^{within} are within-technology elasticities, while σ^{menu} and ζ^{menu} regulate substitution across technologies. σ^{total} and ζ^{total} incorporate adjustments on both margins. For example, for a fixed technology, σ^{within} is the response of i's capital-labor ratio to relative factor prices, $\frac{K_i}{L_i} = \left(\frac{A_i}{B_i}\right)^{\sigma^{within}-1} \left(\frac{r}{w}\right)^{-\sigma^{within}}$. Given any particular choice of factor inputs, i's choice of technologies will satisfy $\left(\frac{A_i K_i}{B_i L_i}\right)^{\frac{\sigma^{within}-1}{\sigma^{within}}} = \left(\frac{A_i/\mathcal{A}_i}{B_i/\mathcal{B}_i}\right)^{\sigma^{menu}-1}$. Together, these imply that the plant's capital-labor ratio will be $\frac{K_i}{L_i} = \left(\frac{\mathcal{A}_i}{\mathcal{B}_i}\right)^{\sigma^{total}-1} \left(\frac{r}{w}\right)^{-\sigma^{total}}$.

With our methodology we are unable to distinguish between the within-technology and technology-menu margins of adjustment, as our cross-sectional estimates capture changes along both. Fortunately, distinguishing between the two margins is not necessary to build up to a long-run aggregate elasticity; we *want* to capture both margins. (J.12) can be used as the starting point for Section 2, which means that our baseline strategy can be used without modification to recover the aggregate elasticity.

While this example is designed so that the envelope of i's technology menu takes a simple nested CES form, we can apply the arguments of Section C.2 and characterize the local elasticity of the envelope of any sufficiently smooth menu of technologies. In particular, if plant i can access a menu of technologies $\mathcal{T}_i$, and each technology $\tau \in \mathcal{T}_i$ is a production function $G_\tau(K, L, M)$, then the envelope of i's technology menu is the reduced form production function

$$F_i(K, L, M) = \max_{\tau \in \mathcal{T}_i} G_\tau(K, L, M).$$

Our estimates correspond to the properties of F_i. In addition, we further extend the argument to a setting in which each plant faces a menu with a discrete set of technologies in Web Appendix J.3.2. In that setting, a plant's factor intensity may jump discretely in response to a marginal change in factor prices if it switches technologies. Under an additional assumption that there is not an atom of plants at the margin between two technologies, we show that the same logic prevails: our baseline approach recovers the aggregate elasticity.

J.3.1 Smooth, Parametric Technology Choice

Given input choices K_i, L_i, and M_i, consider the technology choice problem

$$\max_{A_i, B_i, C_i} \left\{ \left[(A_i K_i)^{\frac{\sigma^{within}-1}{\sigma^{within}}} + (B_i L_i)^{\frac{\sigma^{within}-1}{\sigma^{within}}} \right]^{\frac{\sigma^{within}}{\sigma^{within}-1} \frac{\zeta^{within}-1}{\zeta^{within}}} + (C_i M_i)^{\frac{\zeta^{within}-1}{\zeta^{within}}} \right\}^{\frac{\zeta^{within}}{\zeta^{within}-1}}$$

subject to

$$\left\{ \left[(A_i/\mathcal{A}_i)^{1-\sigma^{menu}} + (B_i/\mathcal{B}_i)^{1-\sigma^{menu}} \right]^{\frac{1-\zeta^{menu}}{1-\sigma^{menu}}} + (C_i/\mathcal{C}_i)^{1-\zeta^{menu}} \right\}^{\frac{1}{1-\zeta^{menu}}} \leq 1 \tag{J.13}$$

Proposition J.1 *The maximized value of output is*

$$\left\{ \left[(\mathcal{A}_i K_i)^{\frac{\sigma^{total}-1}{\sigma^{total}}} + (\mathcal{B}_i L_i)^{\frac{\sigma^{total}-1}{\sigma^{total}}} \right]^{\frac{\sigma^{total}}{\sigma^{total}-1} \frac{\zeta^{total}-1}{\zeta^{total}}} + (\mathcal{C}_i M_i)^{\frac{\zeta^{total}-1}{\zeta^{total}}} \right\}^{\frac{\zeta^{total}}{\zeta^{total}-1}} \tag{J.14}$$

where σ^{total} and ζ^{total} are defined to satisfy

$$\begin{aligned} \frac{1}{\sigma^{total}-1} &= \frac{1}{\sigma^{within}-1} + \frac{1}{\sigma^{menu}-1} \\ \frac{1}{\zeta^{total}-1} &= \frac{1}{\zeta^{within}-1} + \frac{1}{\zeta^{menu}-1} \end{aligned}$$

Proof. Let λ_i be the multiplier on (J.13). Multiplying the FOCs for A_i, B_i, and C_i respectively by K_i, L_i, and M_i and imposing that (J.13) binds gives

$$\left[(A_iK_i)^{\frac{\sigma^{within}-1}{\sigma^{within}}}+(B_iL_i)^{\frac{\sigma^{within}-1}{\sigma^{within}}}\right]^{\frac{\sigma^{within}}{\sigma^{within}-1}\frac{\zeta^{within}-1}{\zeta^{within}}-1}(A_iK_i)^{\frac{\sigma^{within}-1}{\sigma^{within}}}=$$

$$\lambda_i\left[(A_i/\mathcal{A}_i)^{1-\sigma^{menu}}+(B_i/\mathcal{B}_i)^{1-\sigma^{menu}}\right]^{\frac{1-\zeta^{menu}}{1-\sigma^{menu}}-1}(A_i/\mathcal{A}_i)^{1-\sigma^{menu}}$$

$$\left[(A_iK_i)^{\frac{\sigma^{within}-1}{\sigma^{within}}}+(B_iL_i)^{\frac{\sigma^{within}-1}{\sigma^{within}}}\right]^{\frac{\sigma^{within}}{\sigma^{within}-1}\frac{\zeta^{within}-1}{\zeta^{within}}-1}(B_iL_i)^{\frac{\sigma^{within}-1}{\sigma^{within}}}=$$

$$\lambda_i\left[(A_i/\mathcal{A}_i)^{1-\sigma^{menu}}+(B_i/\mathcal{B}_i)^{1-\sigma^{menu}}\right]^{\frac{1-\zeta^{menu}}{1-\sigma^{menu}}-1}(B_i/\mathcal{B}_i)^{1-\sigma^{menu}}$$

$$(C_iM_i)^{\frac{\zeta^{within}-1}{\zeta^{within}}}=\lambda_i\,(C_i/\mathcal{C}_i)^{1-\zeta^{menu}}$$

Adding these together yields $\lambda_i=1$. With this, the FOC for C_i can be expressed as

$$\begin{aligned}(C_iM_i)^{\frac{\zeta^{within}-1}{\zeta^{within}}} &= (\mathcal{C}_iM_i)^{\frac{\frac{\zeta^{within}-1}{\zeta^{within}}(\zeta^{menu}-1)}{\frac{\zeta^{within}-1}{\zeta^{within}}+\zeta^{menu}-1}}=(\mathcal{C}_iM_i)^{\frac{1}{\frac{1}{(\zeta^{menu}-1)}+\frac{\zeta^{within}}{\zeta^{within}-1}}}\\ &= (\mathcal{C}_iM_i)^{\frac{1}{\frac{1}{(\zeta^{menu}-1)}+\frac{1}{\zeta^{within}-1}+1}}=(\mathcal{C}_iM_i)^{\frac{1}{\frac{1}{\zeta^{total}-1}+1}}\\ &= (\mathcal{C}_iM_i)^{\frac{\zeta^{total}-1}{\zeta^{total}}}\end{aligned}\tag{J.15}$$

While the FOCs for A_i and B_i imply the following two equations

$$\begin{aligned}\left(\frac{A_iK_i}{B_iL_i}\right)^{\frac{\sigma^{within}-1}{\sigma^{within}}} &= \left(\frac{A_i/\mathcal{A}_i}{B_i/\mathcal{B}_i}\right)^{1-\sigma^{menu}}\\ \left[(A_iK_i)^{\frac{\sigma^{within}-1}{\sigma^{within}}}+(B_iL_i)^{\frac{\sigma^{within}-1}{\sigma^{within}}}\right]^{\frac{\sigma^{within}}{\sigma^{within}-1}\frac{\zeta^{within}-1}{\zeta^{within}}} &= \left[(A_i/\mathcal{A}_i)^{1-\sigma^{menu}}+(B_i/\mathcal{B}_i)^{1-\sigma^{menu}}\right]^{\frac{1-\zeta^{menu}}{1-\sigma^{menu}}}\end{aligned}$$

The first implies that $\left(\frac{A_iK_i}{B_iL_i}\right)^{\frac{\sigma^{within}-1}{\sigma^{within}}+\sigma^{menu}-1}=\left(\frac{\mathcal{A}_iK_i}{\mathcal{B}_iL_i}\right)^{\sigma^{menu}-1}$, which, along with the definition of σ^{total} implies

$$\left(\frac{A_iK_i}{B_iL_i}\right)^{\frac{\sigma^{within}-1}{\sigma^{within}}}=\left(\frac{\mathcal{A}_iK_i}{\mathcal{B}_iL_i}\right)^{\frac{\frac{\sigma^{within}-1}{\sigma^{within}}(\sigma^{menu}-1)}{\frac{\sigma^{within}-1}{\sigma^{within}}+\sigma^{menu}-1}}=\left(\frac{\mathcal{A}_iK_i}{\mathcal{B}_iL_i}\right)^{\frac{\sigma^{total}-1}{\sigma^{total}}}\tag{J.16}$$

Defining $\rho=\frac{\zeta^{within}-1}{\zeta^{within}}\frac{1}{1-\zeta^{menu}}$, for shorthand, we can rewrite the second as

$$\left[(A_iK_i)^{\frac{\sigma^{within}-1}{\sigma^{within}}}+(B_iL_i)^{\frac{\sigma^{within}-1}{\sigma^{within}}}\right]^{\frac{\sigma^{within}}{\sigma^{within}-1}\rho}=\left[(A_i/\mathcal{A}_i)^{1-\sigma^{menu}}+(B_i/\mathcal{B}_i)^{1-\sigma^{menu}}\right]^{\frac{1}{1-\sigma^{menu}}}$$

Factoring out the terms with B_iL_i from each side, we have

$$\left[\left(\frac{A_iK_i}{B_iL_i}\right)^{\frac{\sigma^{within}-1}{\sigma^{within}}}+1\right]^{\frac{\sigma^{within}}{\sigma^{within}-1}\rho}(B_iL_i)^\rho=\left[\left(\frac{A_i/\mathcal{A}_i}{B_i/\mathcal{B}_i}\right)^{1-\sigma^{menu}}+1\right]^{\frac{1}{1-\sigma^{menu}}}(B_i/\mathcal{B}_i)$$

Using (J.16) and $\frac{1}{1-\sigma^{menu}}=-\frac{\sigma^{total}}{\sigma^{total}-1}+\frac{\sigma^{within}}{\sigma^{within}-1}$, this can be rearranged as

$$\begin{aligned}\left[\left(\frac{\mathcal{A}_iK_i}{\mathcal{B}_iL_i}\right)^{\frac{\sigma^{total}-1}{\sigma^{total}}}+1\right]^{\frac{\sigma^{within}}{\sigma^{within}-1}\rho}(B_iL_i)^\rho &= \left[\left(\frac{\mathcal{A}_iK_i}{\mathcal{B}_iL_i}\right)^{\frac{\sigma^{total}-1}{\sigma^{total}}}+1\right]^{\frac{1}{1-\sigma^{menu}}}\left(\frac{B_iL_i}{\mathcal{B}_iL_i}\right)\\ &= \left[\left(\frac{\mathcal{A}_iK_i}{\mathcal{B}_iL_i}\right)^{\frac{\sigma^{total}-1}{\sigma^{total}}}+1\right]^{-\frac{\sigma^{total}}{\sigma^{total}-1}+\frac{\sigma^{within}}{\sigma^{within}-1}}\left(\frac{B_iL_i}{\mathcal{B}_iL_i}\right)\end{aligned}$$

Collecting the terms with B_iL_i on the left hand side, this can be rearranged as

$$(B_iL_i)^{\frac{\sigma^{within}-1}{\sigma^{within}}}=\left[(\mathcal{A}_iK_i)^{\frac{\sigma^{total}-1}{\sigma^{total}}}+(\mathcal{B}_iL_i)^{\frac{\sigma^{total}-1}{\sigma^{total}}}\right]^{\frac{\sigma^{within}-1}{\sigma^{within}}\frac{\sigma^{total}}{\sigma^{total}-1}\frac{1}{1-\rho}-1}(\mathcal{B}_iL_i)^{\frac{\sigma^{total}-1}{\sigma^{total}}}$$

An analogous argument gives

$$(A_iK_i)^{\frac{\sigma^{within}-1}{\sigma^{within}}}=\left[(\mathcal{A}_iK_i)^{\frac{\sigma^{total}-1}{\sigma^{total}}}+(\mathcal{B}_iL_i)^{\frac{\sigma^{total}-1}{\sigma^{total}}}\right]^{\frac{1}{1-\rho}\frac{\sigma^{within}-1}{\sigma^{within}}\frac{\sigma^{total}}{\sigma^{total}-1}-1}(\mathcal{A}_iK_i)^{\frac{\sigma^{total}-1}{\sigma^{total}}}$$

Summing these two implies

$$(A_iK_i)^{\frac{\sigma^{within}-1}{\sigma^{within}}}+(B_iL_i)^{\frac{\sigma^{within}-1}{\sigma^{within}}}=\left[(\mathcal{A}_iK_i)^{\frac{\sigma^{total}-1}{\sigma^{total}}}+(\mathcal{B}_iL_i)^{\frac{\sigma^{total}-1}{\sigma^{total}}}\right]^{\frac{1}{1-\rho}\frac{\sigma^{within}-1}{\sigma^{within}}\frac{\sigma^{total}}{\sigma^{total}-1}}$$

Or, more simply,

$$\left[(A_iK_i)^{\frac{\sigma^{within}-1}{\sigma^{within}}}+(B_iL_i)^{\frac{\sigma^{within}-1}{\sigma^{within}}}\right]^{\frac{\sigma^{within}}{\sigma^{within}-1}}=\left[(\mathcal{A}_iK_i)^{\frac{\sigma^{total}-1}{\sigma^{total}}}+(\mathcal{B}_iL_i)^{\frac{\sigma^{total}-1}{\sigma^{total}}}\right]^{\frac{\sigma^{total}}{\sigma^{total}-1}\frac{1}{1-\rho}} \quad \text{(J.17)}$$

The definition $\rho\equiv\frac{\zeta^{within}-1}{\zeta^{within}}\frac{1}{1-\zeta^{menu}}$ implies that

$$\frac{\frac{\zeta^{within}-1}{\zeta^{within}}}{1-\rho}=\frac{\frac{\zeta^{within}-1}{\zeta^{within}}}{1-\frac{\zeta^{within}-1}{\zeta^{within}}\frac{1}{1-\zeta^{menu}}}=\frac{1}{\frac{\zeta^{within}}{\zeta^{within}-1}-\frac{1}{1-\zeta^{menu}}}=\frac{1}{1+\frac{1}{\zeta^{within}-1}+\frac{1}{\zeta^{menu}-1}}=\frac{1}{1+\frac{1}{\zeta^{total}-1}}=\frac{\zeta^{total}-1}{\zeta^{total}}$$

which together (J.17) implies

$$\left[(A_iK_i)^{\frac{\sigma^{within}-1}{\sigma^{within}}}+(B_iL_i)^{\frac{\sigma^{within}-1}{\sigma^{within}}}\right]^{\frac{\sigma^{within}}{\sigma^{within}-1}\frac{\zeta^{within}-1}{\zeta^{within}}}=\left[(\mathcal{A}_iK_i)^{\frac{\sigma^{total}-1}{\sigma^{total}}}+(\mathcal{B}_iL_i)^{\frac{\sigma^{total}-1}{\sigma^{total}}}\right]^{\frac{\sigma^{total}}{\sigma^{total}-1}\frac{\zeta^{total}-1}{\zeta^{total}}}$$

This together with (J.15) implies that the maximized value of output is (J.14).

■

J.3.2 Discrete Technology Menu

In this section, we assume that each plant faces a discrete menu of technologies. When a plant switches technologies, its factor intensity may jump discretely, altering the aggregate capital-labor ratio. To simplify the exposition, we build on the simple example of Section 2.1 in which a single industry is composed of a continuum of plants that do not use intermediate inputs.

Plants choose between a finite set of technology families $j = 1, ..., J$, where each technology family consists of a constant returns to scale production function. When plant i uses technology j, its unit cost function is $C_{ij}(r, w)$.[25] Because plant i's production possibilities are the upper envelope of these production functions, its unit cost is $C_i(r, w) = \min_j C_{ij}(r, w)$.

For relative price $\omega = w/r$, $\mathrm{j}_i(\omega)$ is i's choice of technology at ω. Abusing notation, let $\alpha_{ij}(\omega) \equiv \frac{C_{ijr}(1,\omega)}{C_{ij}(1,\omega)}$ be the plant's choice of capital share under technology j and $\alpha_i(\omega) = \alpha_{i\mathrm{j}_i(\omega)}(\omega)$ be plant i's actual capital share given its optimal technology choice. Similarly, let $\theta_i(\omega)$ be i's share of industry expenditures on capital and labor, given its optimal choice of capital and labor as well as the optimal choices of other plants. Finally, let $\sigma_{ij}(\omega) \equiv \frac{C_{ij}(1,\omega)C_{ijrw}(1,\omega)}{C_{ijr}(1,\omega)C_{ijw}(1,\omega)}$ be the local elasticity of substitution of technology j for firm i.

As before, the aggregate capital share is $\alpha(\omega) = \int_i \alpha_i(\omega)\theta_i(\omega)\mathrm{d}i$. The aggregate elasticity of substitution is:

$$\sigma^{agg}(\omega) = \frac{1}{\alpha(\omega)[1-\alpha(\omega)]}\frac{\mathrm{d}\alpha(\omega)}{\mathrm{d}\ln\omega} = \frac{1}{\alpha(\omega)[1-\alpha(\omega)]}\lim_{\omega'\to\omega}\int\left[\alpha_i(\omega')\theta_i(\omega') - \alpha_i(\omega)\theta_i(\omega)\right]\mathrm{d}i$$

We show below in Web Appendix J.3.3 that (suppressing the argument ω)

$$\sigma^{agg} = (1-\chi)\bar{\sigma} + \chi\varepsilon \tag{J.18}$$

where $\bar{\sigma} = \int \frac{\alpha_i(1-\alpha_i)}{\int \alpha_{i'}(1-\alpha_{i'}\mathrm{d}i')}\sigma_i \mathrm{d}i$ and i's local elasticity σ_i is defined as

$$\sigma_i \equiv \sigma_{i\mathrm{j}_i} + \sum_{j'\neq \mathrm{j}_i}\delta\left(\ln\frac{C_i}{C_{ij'}}\right)\frac{\left(\alpha_{ij'}-\alpha_i\right)^2}{\alpha_i(1-\alpha_i)}$$

where $\sigma_{i\mathrm{j}_i}$ is the elasticity of substitution of the technology that i uses and $\delta(\cdot)$ is the Dirac delta function.

σ_i incorporates two types of adjustment. A plant may keep the same technology and adjust factor shares smoothly, or it may switch technologies, in which case its factor shares might jump discontinuously. This jump is captured by the Dirac delta function. Since the measure of plants that might jump has measure zero, the total impact of these jumps is of the same order of magnitude as the cumulative impact of all plants adjusting smoothly.[26]

Next, we note that an estimate of $\bar{\sigma}$ derived from (C.1) in Section C.2, the average plant-level elasticity when elasticities are heterogeneous across plants and defined locally, incorporates

[25] As simple examples, plants may differ in their productivity when using a technology, and these productivity differences could be Hicks-neutral ($C_{ij}(r, w) = \frac{1}{q_{ij}}\tilde{C}_j(r, w)$), factor- augmenting ($C_{ij}(r, w) = \tilde{C}_j(r/A_{ij}, w/B_{ij})$), or non-neutral in an arbitrary way ($C_{ij}(r, w) = \tilde{C}_j(r, w; q_{i,j})$).

[26] When plant i switches technologies, there is no corresponding jump in θ_i, its expenditure on capital and labor, because its marginal cost does not jump; this is simply a manifestation of Berge's Maximum Theorem.

the discontinuous response of factor shares when plants switch technologies. As before, the coefficient delivers an estimate (up to weighting of observations) of how the average capital share responds to changes in factor prices. With technology choice, the estimator recovers $\frac{\mathrm{d}}{\mathrm{d}\ln\omega}\mathbb{E}[\alpha_i] = \lim_{\Delta\to 0}\int[\alpha_i(\omega') - \alpha_i(\omega)]\,\mathrm{d}i$. In Web Appendix J.3.3 below, we show that this is

$$\frac{\mathrm{d}}{\mathrm{d}\ln\omega}\mathbb{E}[\alpha_i] = \frac{\mathrm{d}\left(\frac{1}{I}\int_i \alpha_i(\omega)\mathrm{d}i\right)}{\mathrm{d}\ln\omega} = \frac{1}{I}\int_i \alpha_i(1-\alpha_i)(\sigma_i(\omega)-1)\mathrm{d}i. \tag{J.19}$$

As a consequence, the strategy that computes the aggregate elasticity using the estimate from Section C.2 can also be used without modification to recover the aggregate elasticity in this context. With technology choice, a plant's production function may be discontinuous because it is the envelope of several technology-specific production functions. However, this discontinuity does not add any additional economic forces: aggregate substitution is still determined by within-plant substitution and how consumers substitute across plants in response to their changing price. The only complication is that a plant's factor shares may jump, so additional notation is needed to characterize the within-plant substitution. We have shown that, given our estimation strategy, this distinction is not an important one.

J.3.3 Proofs for Discrete Technology Menu

Let $I(j;\omega)$ be the set of plants that would choose technology j if factor prices were ω. By extension, let $I(j,j';\omega,\omega')$ be the set of plants for whom technology j would be optimal if factor prices were ω and j' would be optimal if factor prices were ω'. We are interested in the derivative $\frac{\mathrm{d}\alpha(\omega)}{\mathrm{d}\ln\omega} = \lim_{\omega'\to\omega}\frac{\alpha(\omega')-\alpha(\omega)}{\omega'/\omega-1}$. Since $\alpha(\omega) = \int \alpha_i(\omega)\theta_i(\omega)\mathrm{d}i$, we can rearrange this as

$$\begin{aligned}\frac{\mathrm{d}\alpha(\omega)}{\mathrm{d}\ln\omega} &= \lim_{\omega'\to\omega}\frac{1}{\omega'/\omega-1}\int_i\left[\alpha_i(\omega')\theta_i(\omega') - \alpha_i(\omega)\theta_i(\omega)\right]\mathrm{d}i \\ &= \lim_{\omega'\to\omega}\int_i \frac{\alpha_i(\omega')-\alpha_i(\omega)}{\omega'/\omega-1}\theta(h_i,\omega)\mathrm{d}i + \lim_{\omega'\to\omega}\int_i \alpha_i(\omega')\frac{\theta_i(\omega')-\theta_i(\omega)}{\omega'/\omega-1}\mathrm{d}i\end{aligned}$$

Lemma J.1

$$\lim_{\omega'\to\omega}\int_i \frac{\alpha_i(\omega')-\alpha_i(\omega)}{\omega'/\omega-1}\theta_i(\omega)\mathrm{d}i = \int_i \alpha_i(\omega)(1-\alpha_i(\omega))(\sigma_i(\omega)-1)\theta_i(\omega)\mathrm{d}i$$

where

$$\sigma_i(\omega) \equiv \sigma_{i\mathrm{j}_i(\omega)}(\omega) + \sum_{j'\neq \mathrm{j}_i(\omega)}\delta\left(\ln\frac{C_i(\omega)}{C_{ij'}(\omega)}\right)\frac{\left(\alpha_{ij'}(\omega)-\alpha_i(\omega)\right)^2}{\alpha_i(\omega)(1-\alpha_i(\omega))}$$

Proof. Define $Q_{jj'}(\omega) \equiv \lim_{\omega'\to\omega}\int_{I(j,j';\omega,\omega')}\frac{\alpha_i(\omega')-\alpha_i(\omega)}{\omega'/\omega-1}\theta_i(\omega)\mathrm{d}i$ so that

$$\lim_{\omega'\to\omega}\int_i \frac{\alpha_i(\omega')-\alpha_i(\omega)}{\omega'/\omega-1}\theta_i(\omega)\mathrm{d}i = \sum_{j,j'}Q_{jj'}(\omega).$$

For plants that choose to use technology j both when factor prices are ω and ω', we have

$$\lim_{\omega'\to\omega}\frac{\alpha_{ij}(\omega')-\alpha_{ij}(\omega)}{\omega'/\omega-1} = \alpha_{ij}(\omega)\left[1-\alpha_{ij}(\omega)\right](\sigma_{ij}(\omega)-1)$$

This along with $\lim_{\omega' \to \omega} I(j, j; \omega, \omega') = I(j, \omega)$ yields

$$Q_{jj}(\omega) = \int_{I(j;\omega)} \alpha_i(\omega)(1 - \alpha_i(\omega))(\sigma_{ij}(\omega) - 1)\theta_i(\omega)\mathrm{d}i$$

We next turn to characterizing $Q_{jj'}(\omega)$ for $j \neq j'$. Let $c_{ij}(\omega) \equiv C_{ij}(1, \omega)$. Consider plant $i \in I(j, \omega)$. For that plant to be in the set $I(j, j'; \omega, \omega')$, it must be that j' is its preferred technology when factor prices are ω', i.e., $c_{ij'}(\omega') \leq \min_{j'' \neq j'} c_{ij''}(\omega')$ or equivalently $\ln \frac{\min_{j'' \neq j'} c_{ij''}(\omega')}{c_{ij'}(\omega')} \geq 0$. Letting H be the Heaviside step function, we can express $Q_{jj'}$ as

$$\begin{aligned} Q_{jj'}(\omega) &= \lim_{\omega' \to \omega} \int_{I(j,j';\omega,\omega')} \frac{\alpha_{ij'}(\omega') - \alpha_{ij}(\omega)}{\omega'/\omega - 1} \theta(h_i, \omega)\mathrm{d}i \\ &= \lim_{\omega' \to \omega} \int_{I(j,\omega)} \frac{H\left(\ln \frac{\min_{j'' \neq j'} c_{ij''}(\omega')}{c_{ij'}(\omega')}\right)}{\omega'/\omega - 1} \left[\alpha_{ij'}(\omega') - \alpha_{ij}(\omega)\right] \theta(h_i, \omega)\mathrm{d}i \end{aligned}$$

Using L'Hopital's rule gives

$$Q_{jj'}(\omega) = \int_{I(j;\omega)} \lim_{\omega' \to \omega} H'\left(\ln \frac{\min_{j'' \neq j'} c_{ij''}(\omega')}{c_{ij'}(\omega')}\right) \frac{\mathrm{d} \ln \frac{\min_{j'' \neq j} c_{ij''}(\omega')}{c_{ij'}(\omega')}}{\mathrm{d} \ln \omega'} \left[\alpha_{ij'}(\omega') - \alpha_{ij}(\omega)\right] \theta(h_i, \omega)\mathrm{d}i$$

In the limit as $\omega' \to \omega$, $j = \arg\min_{j'' \neq j'} c_{ij''}(\omega)$. In addition, Shephard's Lemma implies $\frac{\mathrm{d} \ln \frac{c_{ij}(\omega')}{c_{ij'}(\omega')}}{\mathrm{d} \ln \omega'} = (1 - \alpha_{ij}(\omega')) - (1 - \alpha_{ij'}(\omega')) = \alpha_{ij'}(\omega') - \alpha_{ij}(\omega')$. With these, we have

$$Q_{jj'}(\omega) = \int_{I(j;\omega)} H'\left(\ln \frac{c_{ij}(\omega)}{c_{ij'}(\omega)}\right) \left[\alpha_{ij'}(\omega') - \alpha_{ij}(\omega)\right]^2 \theta(h_i, \omega)\mathrm{d}i$$

The desired result follows from summing across j, j' and noting that the derivative of the Heaviside step function is the Dirac delta function. ∎

Lemma J.2

$$\lim_{\omega' \to \omega} \int_i \alpha_i(\omega') \frac{\theta_i(\omega') - \theta_i(\omega)}{\omega'/\omega - 1} \mathrm{d}i = (\varepsilon - 1) \int_i [\alpha_i(\omega) - \alpha(\omega)]^2 \theta_i(\omega)\mathrm{d}i$$

Proof. We can express the limit as

$$\lim_{\omega' \to \omega} \int_i \alpha_i(\omega') \frac{\theta_i(\omega') - \theta_i(\omega)}{\omega'/\omega - 1} \mathrm{d}i = \sum_{j,j'} \lim_{\omega' \to \omega} \int_{I(j,j';\omega,\omega')} \alpha_i(\omega') \frac{\theta_i(\omega') - \theta_i(\omega)}{\omega'/\omega - 1} \mathrm{d}i$$

For plants that choose to use technology j both when factor prices are ω and ω', we have, following the logic of (7),

$$\lim_{\omega' \to \omega} \frac{\theta_{ij}(\omega') - \theta_{ij}(\omega)}{\omega'/\omega - 1} = \left[\alpha_{ij}(\omega) - \alpha(\omega)\right] \theta_{ij}(\omega)(\varepsilon - 1)$$

This along with $\lim_{\omega' \to \omega} I(j, j; \omega, \omega') = I(j, \omega)$ yields

$$\lim_{\omega' \to \omega} \int_{I(j,j;\omega,\omega')} \alpha_i(\omega') \frac{\theta_i(\omega') - \theta_i(\omega)}{\omega'/\omega - 1} \mathrm{d}i = \int_{I(j;\omega)} (\alpha_i(\omega) - \alpha(\omega))^2 (\varepsilon - 1)\theta_i(\omega)\mathrm{d}i$$

For $j \neq j'$, we show that the limit is zero. By Berge's Maximum Theorem, i's unit cost is continuous in ω, and therefore $\theta_i(\omega)$ is continuous in ω. Further, for $i \in I(j, j'; \omega, \omega')$, $\lim_{\omega' \to \omega} \frac{\theta_i(\omega')/\theta_i(\omega)-1}{\omega'/\omega-1}$ is bounded below by $[\min\{\alpha_i(\omega), \alpha_i(\omega')\} - \alpha](\varepsilon-1)$ and bounded above by $[\max\{\alpha_i(\omega), \alpha_i(\omega')\} - \alpha](\varepsilon-1)$. The second term is therefore zero because in the limit, the set $\lim_{\omega' \to \omega} I(j, j'; \omega, \omega')$ has zero measure. ■

Together, Lemma J.1 and Lemma J.2 deliver the following proposition characterizing the aggregate capital-labor elasticity of substitution:

Proposition J.2 *With technology choice, the aggregate elasticity of substitution is (suppressing the argument ω)*

$$\sigma^{agg} = (1-\chi)\bar{\sigma} + \chi\varepsilon$$

where $\bar{\sigma} = \int_i \frac{\alpha_i(1-\alpha_i)}{\int_{i'} \alpha_{i'}(1-\alpha_{i'})} \sigma_i \mathrm{d}i$ and σ_i is defined as in Lemma J.1.

Proof. This follows from the previous two lemmas and the strategy of Appendix A. ■

Proposition J.3

$$\frac{\mathrm{d}\left(\int_i \alpha_i(\omega)\mathrm{d}i\right)}{\mathrm{d}\ln\omega} = \int_i \alpha_i(\omega)(1-\alpha_i(\omega))(\sigma_i(\omega)-1)\mathrm{d}i$$

Proof. The proof follows Lemma J.1 almost exactly, with the only exception that $\theta_i(\omega)$ is replaced by $\frac{1}{I}$. ■

J.4 Local vs. National Elasticities

In this section we examine several reasons why the response of plants' capital-labor ratios to local factor prices might differ from the response to a national change.[27] Our identification strategy has focused on studying how plant capital-labor ratios respond to local factor prices. There are a few reasons why this may differ from plants' response to a nationwide change in the wage. It is, of course, the latter which is relevant for an aggregate elasticity at the national level. While these issues are seldom discussed in the literature, they are relevant for *any* estimate of an elasticity of substitution at a level of aggregation smaller than the entire world.

Sorting

Our estimates do not account for the possibility that plants select locations in response to factor prices. To see why this might matter, consider the following extreme example: Suppose plants cannot adjust their factor usage but can move freely. Then we would expect to find more labor intensive plants in locations with lower wages. A national increase in the wage would not, however, change any plant's factor usage. Thus, to the extent that this channel is important, our estimated elasticity will overstate the true elasticity.

Plants' ability to sort across locations likely varies by industry. We would expect industries in which plants are more mobile to be more clustered in particular areas. This could depend, for example, on how easily goods can be shipped to other locations. Raval (2019) addressed this by looking at a set of ten large four-digit industries located in almost all MSAs and states. These are industries for which we would expect sorting across locations to be least important. The leading example of this is ready-mixed concrete; because concrete cannot be shipped very far, concrete

[27] The distinction has been emphasized recently in the debate about the size of government spending multipliers; see Beraja et al. (forthcoming).

plants exist in every locality. Elasticities for these industries are similar to the estimates for all industries in our baseline, with average elasticities of 0.40 for 1987, 0.51 for 1992, and 0.38 for 1997.

Within-Firm Coordination

In our baseline model, we assumed that each plant independently selects its factor intensity in response to local factor prices. It is possible, however, that a firm that operates plants in many locations might derive some scale economy by operating all of its plants at similar capital-labor ratios. If this is the case, then a change in factor prices in one location would, by altering the choices of these multi-unit firms, affect factor intensities in other locations. A potential problem for our approach is that, in such an environment, comparing factor intensities across locations would not reveal the full extent of substitution in response to factor prices.[28] Thus our estimate would understate the true elasticity. One can gauge the importance of this channel by estimating an elasticity of substitution among the subset of plants that belong to multi-unit firms. If this channel is important, one would expect that plants in multi-unit firms would respond less to their local wages than standalone plants. However, column 4 of Table II indicates that the estimated elasticity among this subset is higher than our OLS estimates, suggesting that this channel is not of first order importance.

The Technological Frontier

A similar issue may arise when we consider changes in the technological frontier. A change in factor prices in one location might induce the creation of intermediates that favor particular technologies. The interpretation of our estimate depends on whether those intermediates are available nationwide (as assumed in Section 4.3), or only locally.

Assessment

One way to gauge the importance of these mechanisms is to ask how factor shares in a location respond to the wage in nearby locations. For example, plants that move are more likely to move to nearby locations. To the extent that moving across locations is important, an decrease in the wage in a nearby location would induce the most labor intensive plant to move to that nearby location, raising the average local capital share. Similarly, a decline in the wage in a nearby location may induce the creation of intermediate inputs that favor capital. To the extent that the intermediates are also available locally, these would also favor capital locally, and lower the average local capital share.

A regression of local factor shares on local wages and the average wages of all other locations in the state can reveal the impact of these mechanisms. Note that it can only reveal the net impact of all of them; we cannot distinguish whether neither mechanism matters or that they both matter but offset each other. Fortunately it is the net effect that is relevant for informing us about the gap between the cross-sectional differences and national changes. When we run this regression, we cannot reject that the net impact of both mechanisms is zero. See Web Appendix C.6 for these results.[29]

[28] As an extreme example, suppose the economy consisted of a single firm that operated in two locations and chose a common capital-labor ratio for its two plants. Our methodology would never uncover differences in capital-labor ratios across locations no matter how much the firm adjusted this common ratio.

[29] As discussed above, such a regressions cannot pick up any induced changes in the technological frontier that are truly global.

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